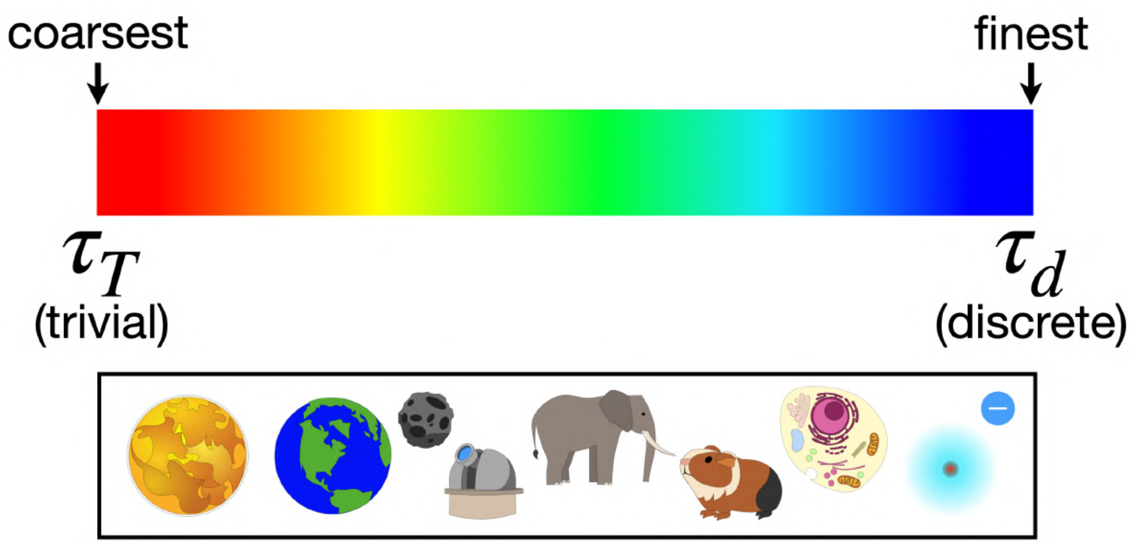
What Makes a Space TOPOLOGICAL?

by DIBEOS



"Topology is precisely the mathematical discipline that allows the passage from the local to the global." – **René Thom**

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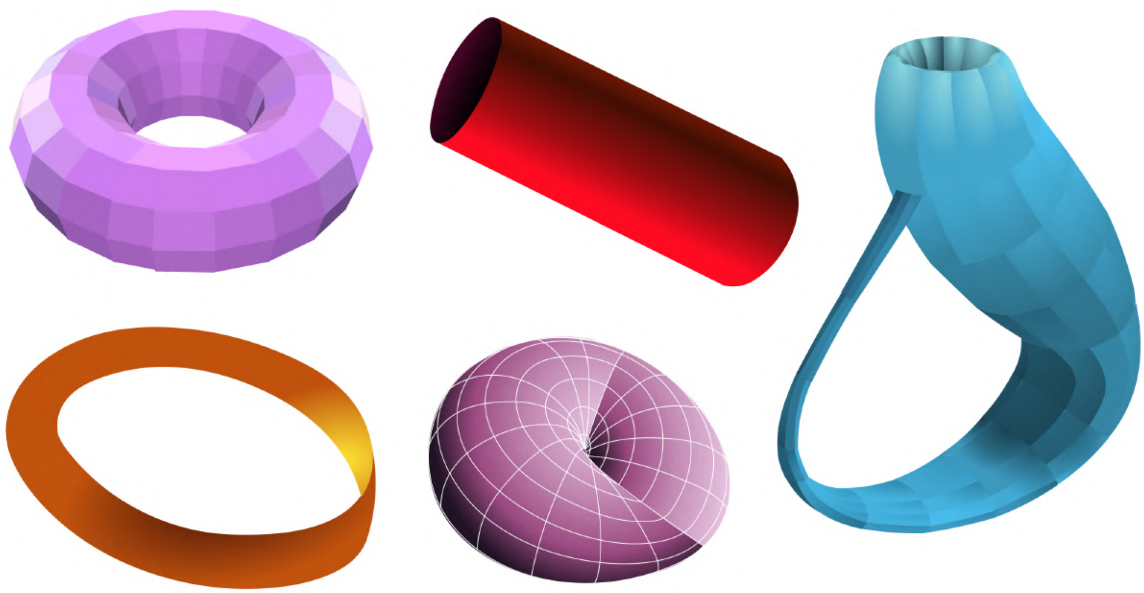
Introduction

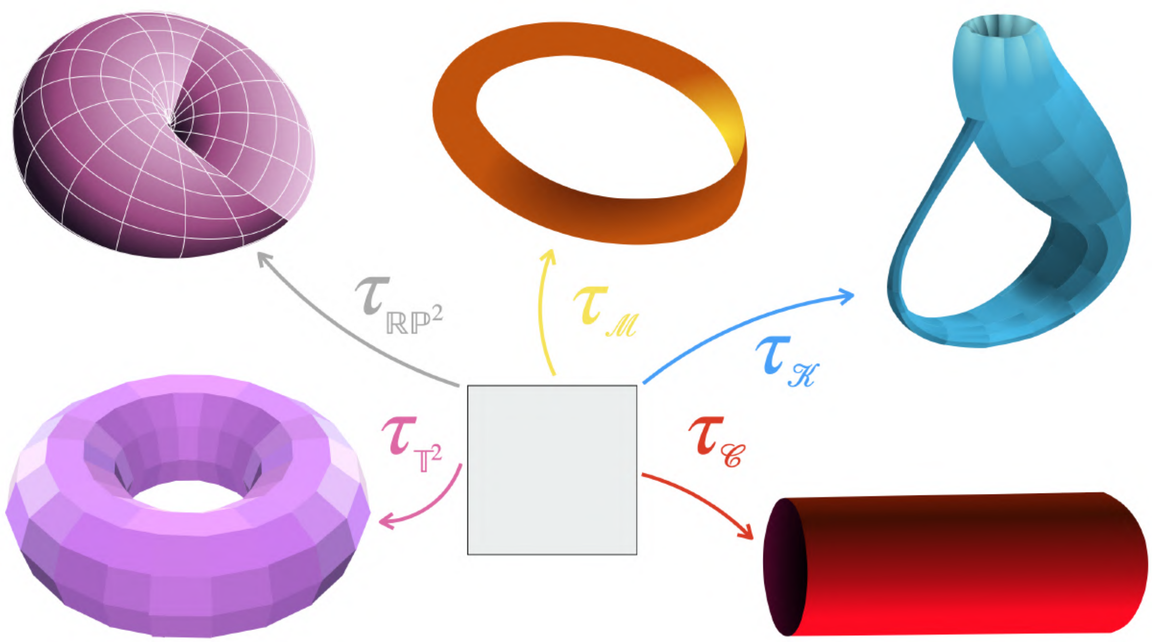
When you think of topology as a whole, usually shapes like these come to mind:



But how can we create them?

All you gotta do is choose the correct topology for each one of them. But what is *a topology* after all? It's a set of *open sets*. Ok... but what does the word *open* mean in this context? Its meaning is much more abstract than you might think.

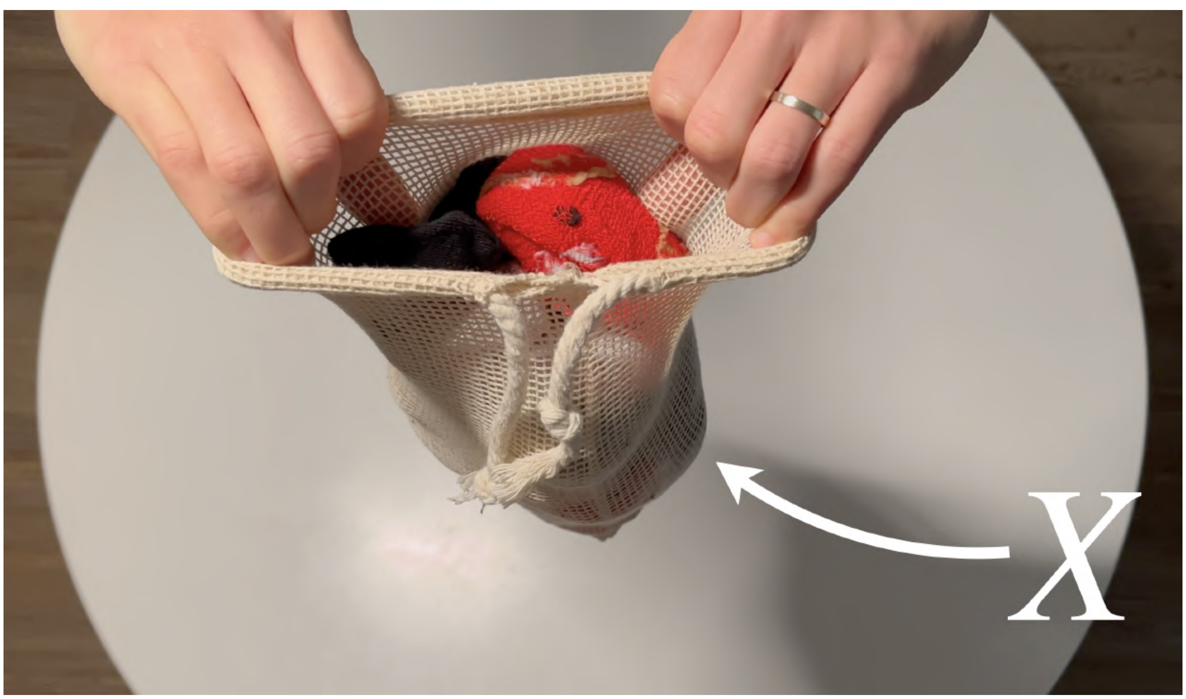
Did you know that all of these shapes can be created from a simple square?



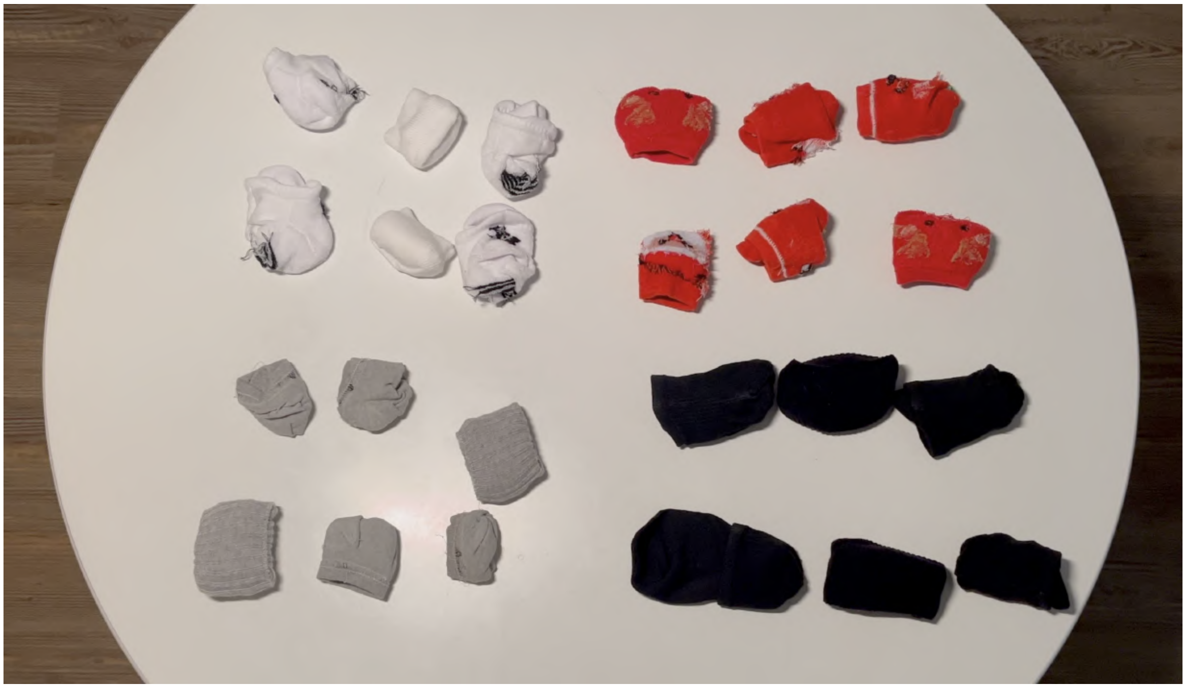
When I was learning these concepts, I couldn't really understand the connection between all these *elastic shapes* and *open sets*. And to be honest, making this PDF and the YouTube video, forced me to research these concepts all over the place. And this really helped me clarify a lot of things! I hope we will be able to do the same for you.

So, let's start with a **bag of socks**.

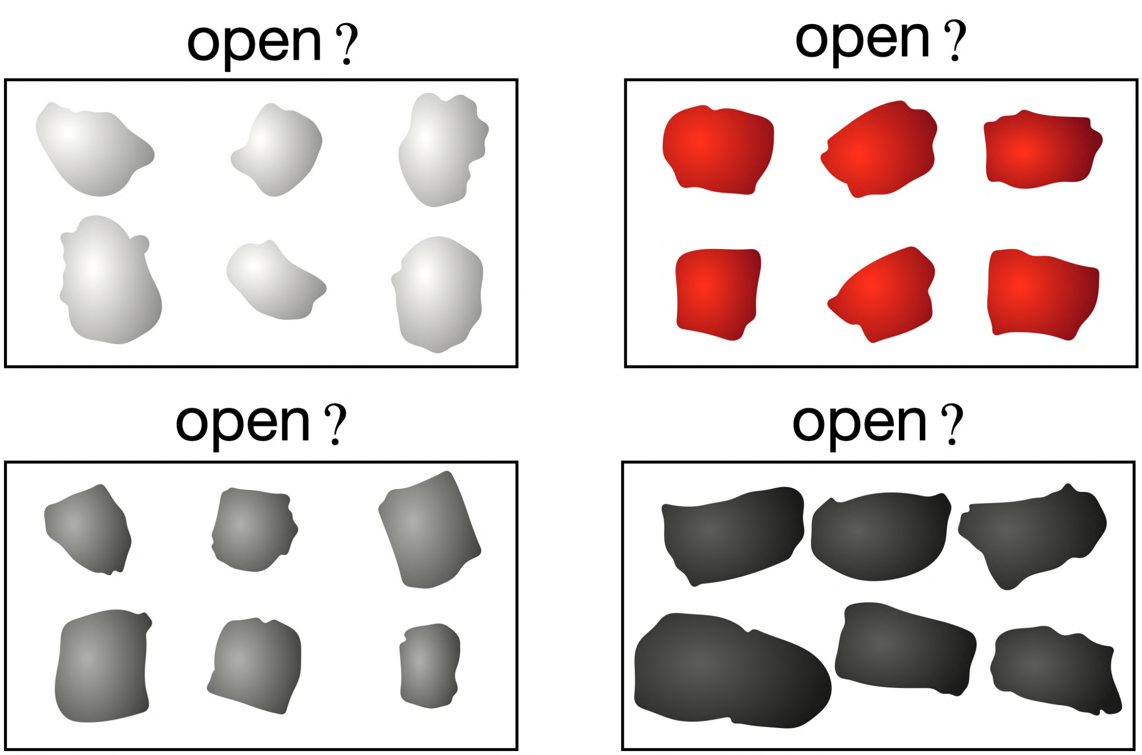
Open Sets



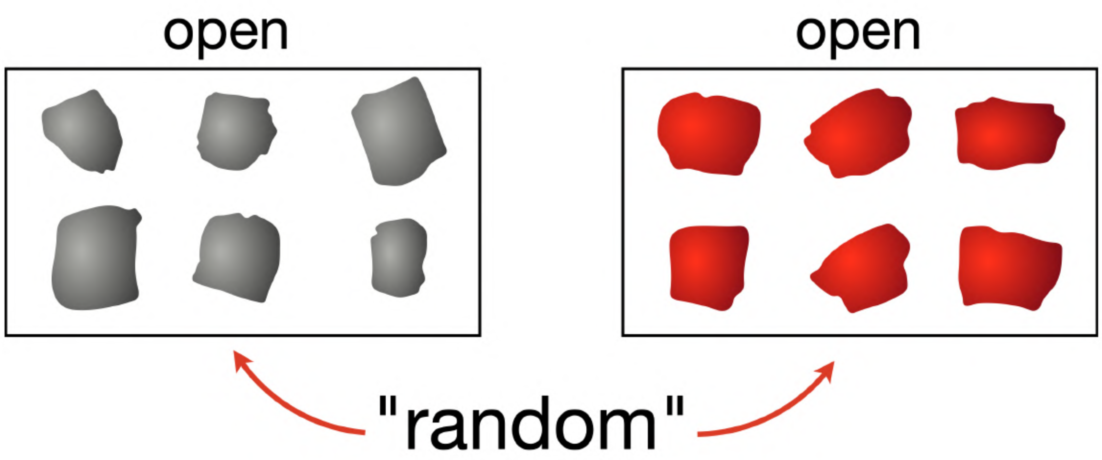
This bag of socks is our set X. We have subsets of *black*, *grey*, *white* and *red* socks.



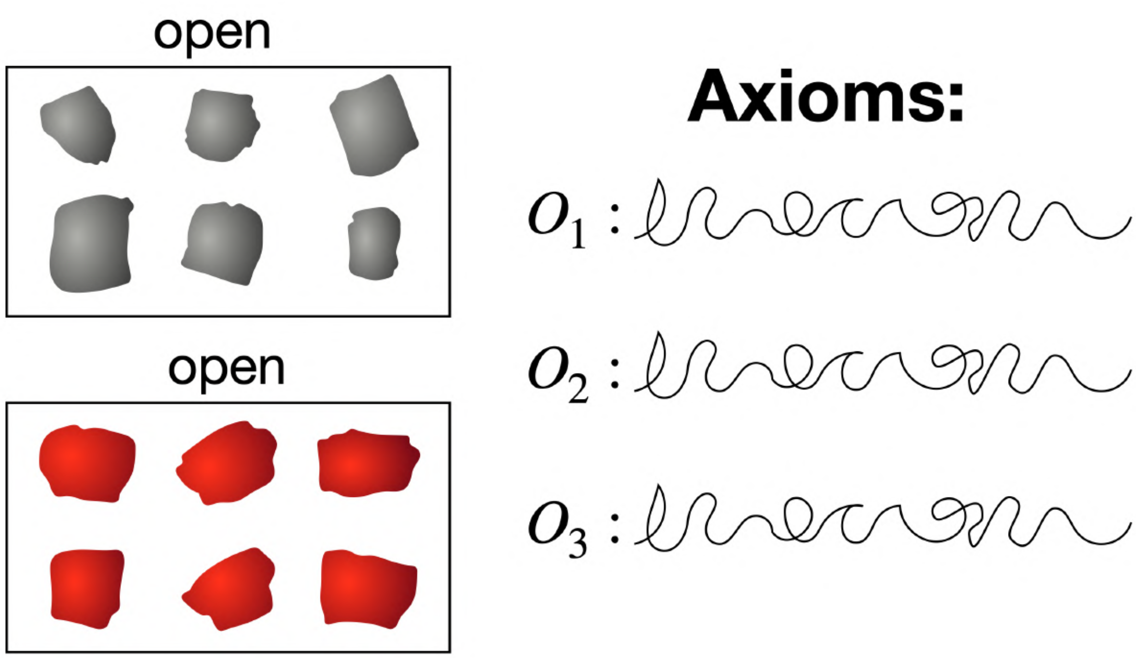
Are any of these *open*?



Well, let's just "randomly" decide that the following two subsets of my bag of socks are going to be called *open* (whatever that means...):

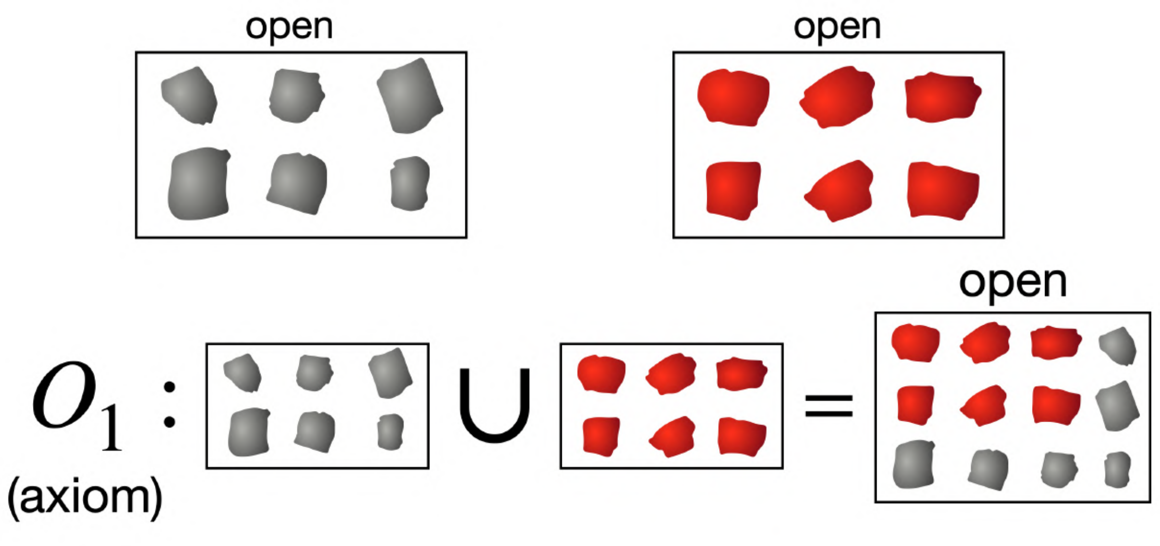


And this is the first step we can take to construct a topology on X. We can start with any collection of subsets of X that we want to call *open*, as long as 3 very important axioms are satisfied down the road.



Let's apply the first one:

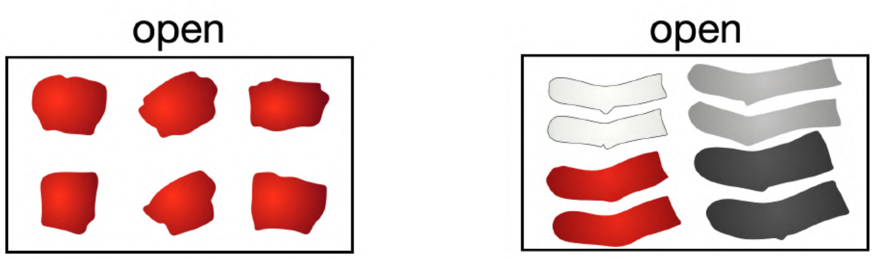
(I) *If the grey and red socks are considered open subsets, then their union must also be considered open.*

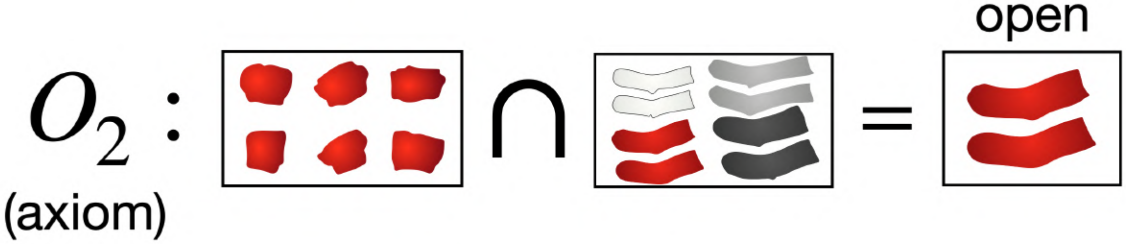


But, when we unroll all the socks we find out a new feature about them. Some are long, and some are short.

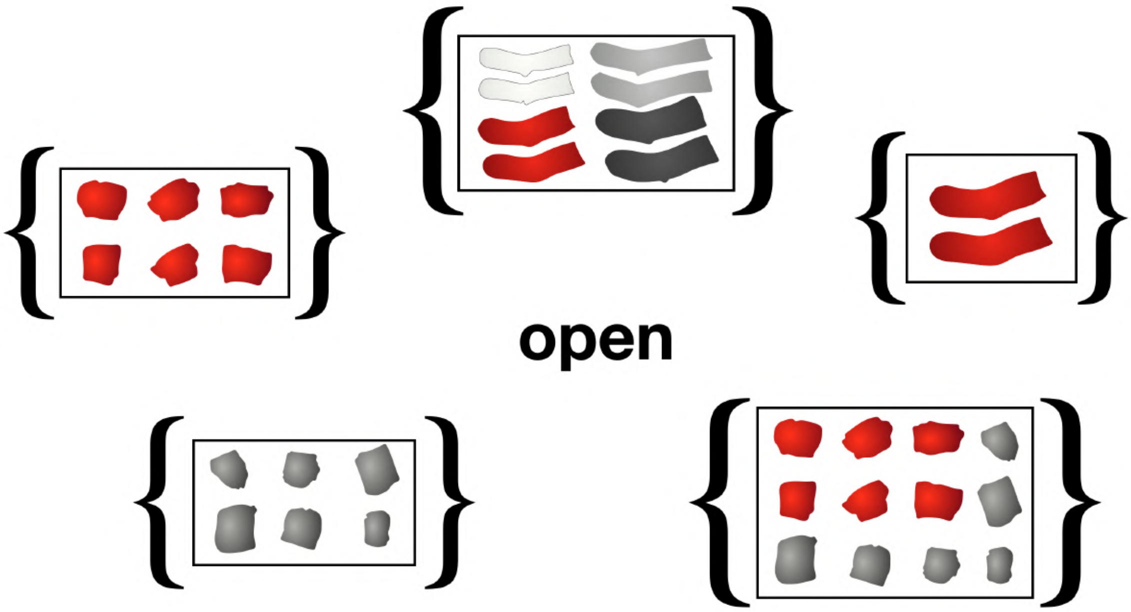
You know what?! Let's make another "random" decision of calling the subset of all long socks *open*, regardless of their color.

Then the second axiom would impose that, for example, the intersection of the subset of red socks with the subset of long socks must also be considered open. **(II)**



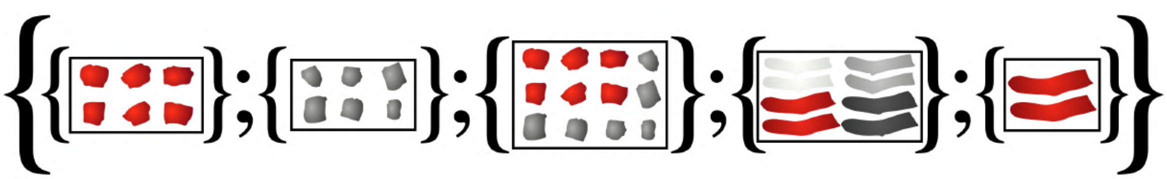


So far, we've decided that some groups of socks are going to be called *open*.

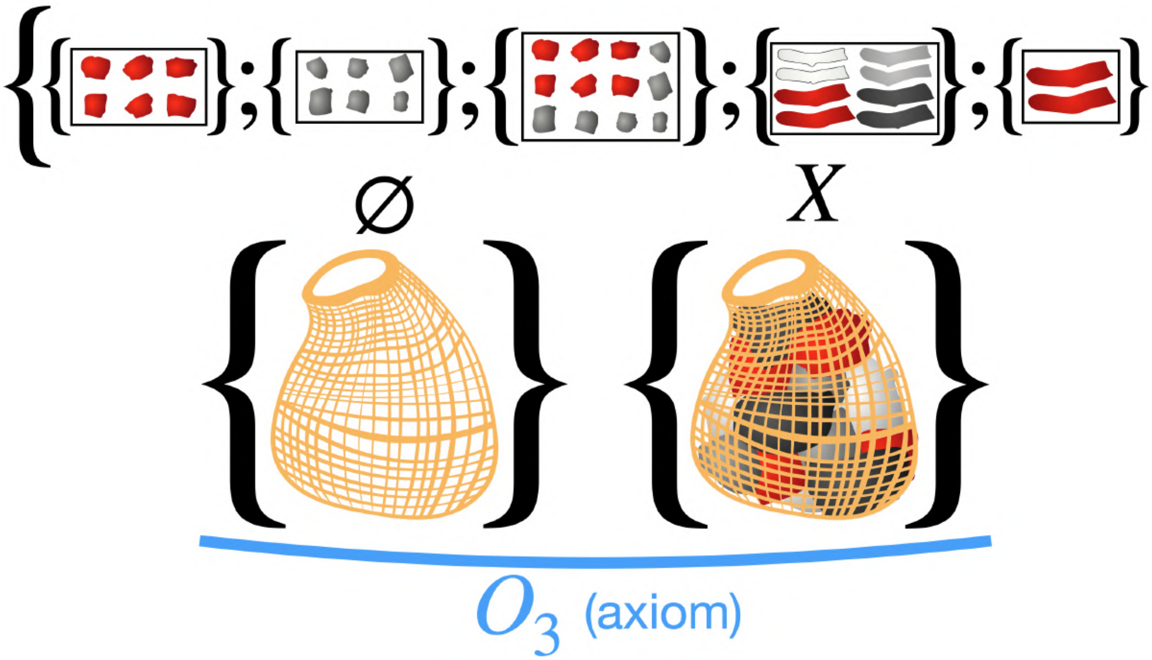


But don't try to assign any physical, or visual, meaning for the word open. All it means here is that these open sets, when put together, satisfy axioms **(I)** and **(II)** that we've just seen.

All of these are open subsets in the bag of socks. Now, if I collect them into a set of sets,

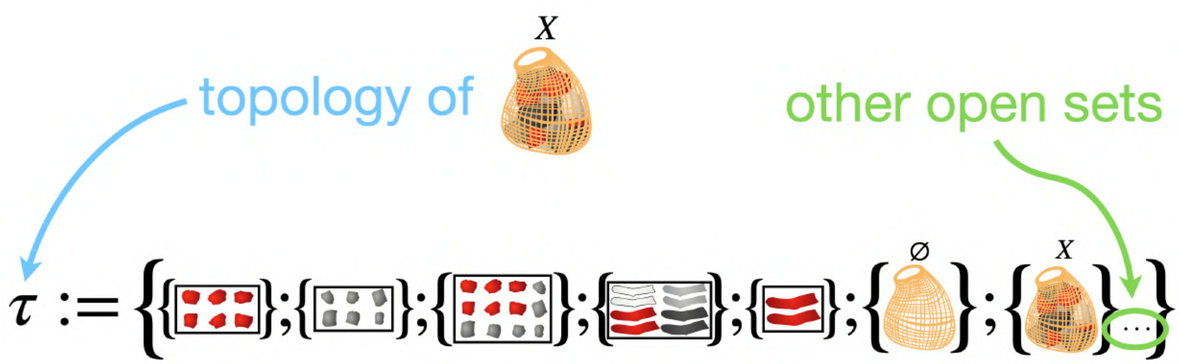


and then I also throw in the empty set $\emptyset$ and the entire bag of socks X **(III)**,



then I've almost got something really special!

This set of sets (together with some other open subsets we will discuss in a moment) is usually denoted with the Greek letter τ (tau):



And this is called **a topology of** X.

Now, things are about to get interesting!!

If you want to have access to the FULL-PDF version, click on this link. We added a few exercises at the end (with detailed solutions) and extra explanations throughout it. Also, I remind you guys that we provide all of our FULL-PDFs for free for all members of the channel. Just join us on YouTube! We'd like to keep our videos free of interruptions and sponsors, so that the sole focus is the subject at hand. But in order to do that we need your help. Thanks for supporting our work.

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