Math Books Don't Know How to Teach

by DiBeos

"The difficulty lies not so much in developing new ideas as in escaping from old ones." – **J. M. Keynes**

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Introduction

You read it once, you read it twice, you read it for the third time (this time even slower), but no matter how much you try, you just can't understand what the theorem is telling you. Then you remember that everybody says that this is a great book to learn **Point-Set Topology**, or **Functional Analysis**, or **Abstract Algebra** for the very first time. The book must be good, right?! Otherwise people would not be suggesting it. So probably you are the one who is just not good enough to understand it.

Well, I don't think you are the problem. I think the books are the problem. And today we will learn a little bit of *Point-Set Topology* the **wrong way**! And right after that, we will learn it the **right way**! And you will be the judge of whether this, that I am about to show you, is the better method or not. Let's get started.

The Wrong Way

"Topology" by James R. Munkres is one of the most classic and widely known books in point-set topology. Let's read **theorem 17.5**, and tell me sincerely whether you understand it or not:

The definition of the closure of a set does not give us a convenient way for actually finding the closures of specific sets, since the collection of all closed sets in X, like the collection of all open sets, is usually much too big to work with. Another way of describing the closure of a set, useful because it involves only a basis for the topology of X, is given in the following theorem.

First let us introduce some convenient terminology. We shall say that a set A ***intersects*** a set B if the intersection $A \cap B$ is not empty.

Theorem 17.5. *Let A be a subset of the topological space X.*

(a) *Then $x \in \bar{A}$ if and only if every open set U containing x intersects A.*

(b) *Supposing the topology of X is given by a basis, then $x \in \bar{A}$ if and only if every basis element B containing x intersects A.*

Proof. Consider the statement in (a). It is a statement of the form $P \Leftrightarrow Q$. Let us transform each implication to its contrapositive, thereby obtaining the logically equivalent statement (not P) $\Leftrightarrow$ (not Q). Written out, it is the following:

$x \notin \bar{A} \Longleftrightarrow$ there exists an open set U containing x that does not intersect A.

In this form, our theorem is easy to prove. If x is not in $\bar{A}$, the set $U = X - \bar{A}$ is an open set containing x that does not intersect A, as desired Conversely, if there exists an open set U containing x which does not intersect A, then $X - U$ is a closed set containing A By definition of the closure $\bar{A}$, the set $X - U$ must contain $\bar{A}$, therefore, x cannot be in $\bar{A}$.

Statement (b) follows readily If every open set containing x intersects A, so does every basis element B containing x, because B is an open set. Conversely, if every basis element containing x intersects A, so does every open set U containing x, because U contains a basis element that contains x. ■

Mathematicians often use some special terminology here. They shorten the statement "U is an open set containing x" to the phrase

"U is a ***neighborhood*** of x."

Using this terminology, one can write the first half of the preceding theorem as follows:

If A is a subset of the topological space X, then $x \in \bar{A}$ if and only if every neighborhood of x intersects A.

EXAMPLE 6 Let X be the real line $\mathbb{R}$. If $A = (0, 1]$, then $\bar{A} = [0, 1]$, for every neighborhood of 0 intersects A, while every point outside $[0, 1]$ has a neighborhood disjoint from A Similar arguments apply to the following subsets of X

If $B = \{1/n \mid n \in \mathbb{Z}_+\}$, then $\bar{B} = \{0\} \cup B$ If $C = \{0\} \cup (1, 2)$, then $\bar{C} = \{0\} \cup [1, 2]$ If $\mathbb{Q}$ is the set of rational numbers, then $\bar{\mathbb{Q}} = \mathbb{R}$ If $\mathbb{Z}_+$ is the set of positive integers, then $\bar{\mathbb{Z}}_+ = \mathbb{Z}_+$. If $\mathbb{R}_+$ is the set of positive reals, then the closure of $\mathbb{R}_+$ is the set $\mathbb{R}_+ \cup \{0\}$. (This is the reason we introduced the notation $\bar{\mathbb{R}}_+$ for the set $\mathbb{R}_+ \cup \{0\}$, back in §2)

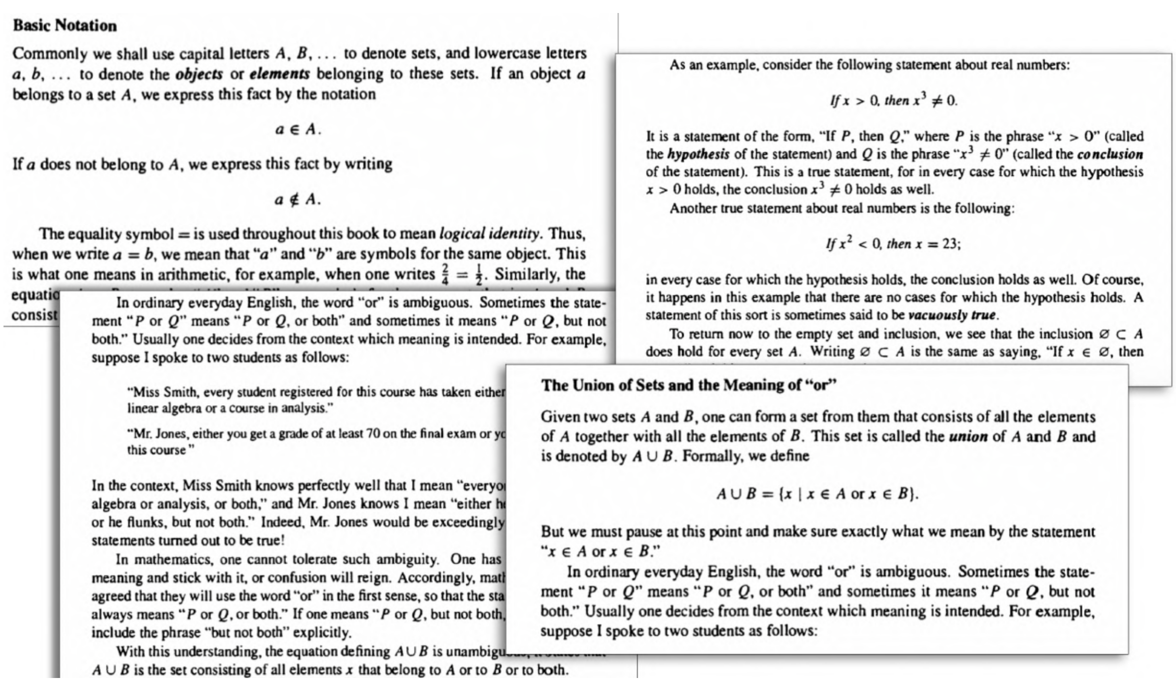
And you can read the proof right below the theorem.

If you never studied point-set topology before, most likely you had a hard time trying to understand what the theorem is all about. If you have already studied point-set topology, please try to remember when you were learning these concepts for the very first time. I'm pretty sure that most of you would not have fully understood what we just read.

Now, a good question would be: *"what are the **prerequisites** for studying this book?"*. Let's see what it says:

> **Prerequisites.** There are no formal subject matter prerequisites for studying most of this book. I do not even assume the reader knows much set theory. Having said that, I must hasten to add that unless the reader has studied a bit of analysis or "rigorous calculus," much of the motivation for the concepts introduced in the first part of the book will be missing. Things will go more smoothly if he or she already has had some experience with continuous functions, open and closed sets, metric spaces, and the like, although none of these is actually assumed. In Part II, we do assume familiarity with the elements of group theory.

Ok, to be fair, when it comes to the first chapter, I would say that he does put in more effort than math books usually would to build the foundation. Although it is still far from good.


Basic Notation

Commonly we shall use capital letters $A, B, \dots$ to denote sets, and lowercase letters $a, b, \dots$ to denote the ***objects*** or ***elements*** belonging to these sets. If an object a belongs to a set A, we express this fact by the notation

$$a \in A.$$

If a does not belong to A, we express this fact by writing

$$a \notin A.$$

The equality symbol = is used throughout this book to mean *logical identity*. Thus, when we write $a = b$, we mean that "a" and "b" are symbols for the same object. This is what one means in arithmetic, for example, when one writes $\frac{2}{4} = \frac{1}{2}$. Similarly, the equatio… consist…

As an example, consider the following statement about real numbers:

$$\text{If } x > 0, \text{ then } x^3 \neq 0.$$

It is a statement of the form, "If P, then Q," where P is the phrase "$x > 0$" (called the ***hypothesis*** of the statement) and Q is the phrase "$x^3 \neq 0$" (called the ***conclusion*** of the statement). This is a true statement, for in every case for which the hypothesis $x > 0$ holds, the conclusion $x^3 \neq 0$ holds as well.

Another true statement about real numbers is the following:

$$\text{If } x^2 < 0, \text{ then } x = 23;$$

in every case for which the hypothesis holds, the conclusion holds as well. Of course, it happens in this example that there are no cases for which the hypothesis holds. A statement of this sort is sometimes said to be ***vacuously true***.

To return now to the empty set and inclusion, we see that the inclusion $\varnothing \subset A$ does hold for every set A. Writing $\varnothing \subset A$ is the same as saying, "If $x \in \varnothing$, then

In ordinary everyday English, the word "or" is ambiguous. Sometimes the statement "P or Q" means "P or Q, or both" and sometimes it means "P or Q, but not both." Usually one decides from the context which meaning is intended. For example, suppose I spoke to two students as follows:

"Miss Smith, every student registered for this course has taken either linear algebra or a course in analysis."

"Mr. Jones, either you get a grade of at least 70 on the final exam or yo… this course"

In the context, Miss Smith knows perfectly well that I mean "everyo… algebra or analysis, or both," and Mr. Jones knows I mean "either h… or he flunks, but not both." Indeed, Mr. Jones would be exceedingly… statements turned out to be true!

In mathematics, one cannot tolerate such ambiguity. One has… meaning and stick with it, or confusion will reign. Accordingly, math… agreed that they will use the word "or" in the first sense, so that the sta… always means "P or Q, or both." If one means "P or Q, but not both,… include the phrase "but not both" explicitly.

With this understanding, the equation defining $A \cup B$ is unambigu… $A \cup B$ is the set consisting of all elements x that belong to A or to B or to both.

The Union of Sets and the Meaning of "or"

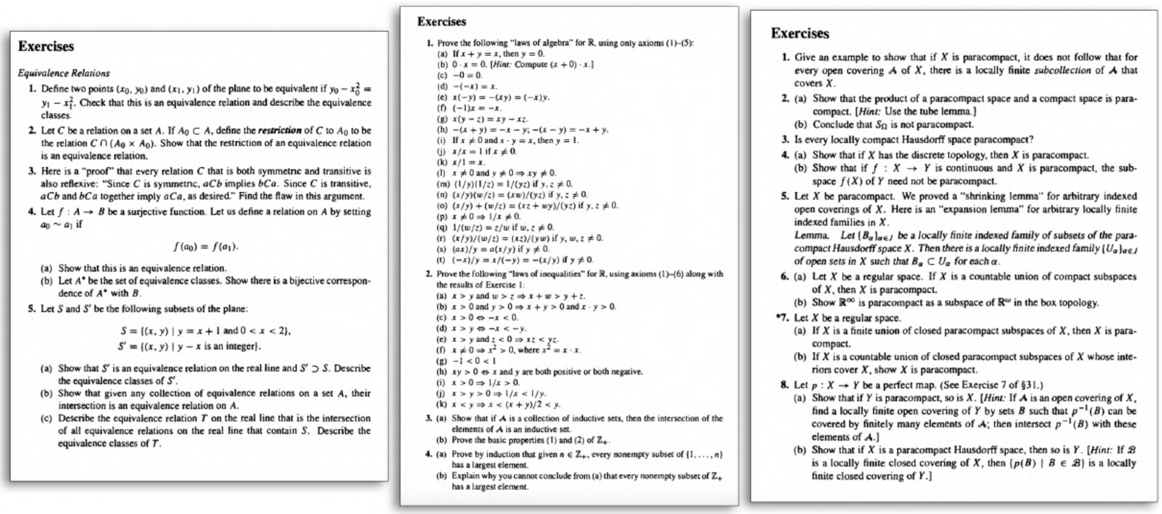
Given two sets A and B, one can form a set from them that consists of all the elements of A together with all the elements of B. This set is called the ***union*** of A and B and is denoted by $A \cup B$. Formally, we define

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}.$$

But we must pause at this point and make sure exactly what we mean by the statement "$x \in A$ or $x \in B$."

In ordinary everyday English, the word "or" is ambiguous. Sometimes the statement "P or Q" means "P or Q, or both" and sometimes it means "P or Q, but not both." Usually one decides from the context which meaning is intended. For example, suppose I spoke to two students as follows:

A beginner will definitely have a very hard time trying to digest the content, especially in later chapters. And another problem is that after building the foundation, the book goes straight into the rigor of the concepts which are completely new to the reader without building any *intuition*. We meet the classic wall of definitions, theorems and proofs. Not to mention the classic problem of exercises without solutions.



Exercises

Equivalence Relations

1. Define two points (x_0, y_0) and (x_1, y_1) of the plane to be equivalent if $y_0 - x_0^2 = y_1 - x_1^2$. Check that this is an equivalence relation and describe the equivalence classes.
2. Let C be a relation on a set A. If $A_0 \subset A$, define the ***restriction*** of C to A_0 to be the relation $C \cap (A_0 \times A_0)$. Show that the restriction of an equivalence relation is an equivalence relation.
3. Here is a "proof" that every relation C that is both symmetric and transitive is also reflexive: "Since C is symmetric, aCb implies bCa. Since C is transitive, aCb and bCa together imply aCa, as desired." Find the flaw in this argument.
4. Let $f : A \to B$ be a surjective function. Let us define a relation on A by setting $a_0 \sim a_1$ if
$$f(a_0) = f(a_1).$$
(a) Show that this is an equivalence relation.
(b) Let A^* be the set of equivalence classes. Show there is a bijective correspondence of A^* with B.
5. Let S and S' be the following subsets of the plane:
$$S = \{(x, y) \mid y = x + 1 \text{ and } 0 < x < 2\},$$
$$S' = \{(x, y) \mid y - x \text{ is an integer}\}.$$
(a) Show that S' is an equivalence relation on the real line and $S' \supset S$. Describe the equivalence classes of S'.
(b) Show that given any collection of equivalence relations on a set A, their intersection is an equivalence relation on A.
(c) Describe the equivalence relation T on the real line that is the intersection of all equivalence relations on the real line that contain S. Describe the equivalence classes of T.

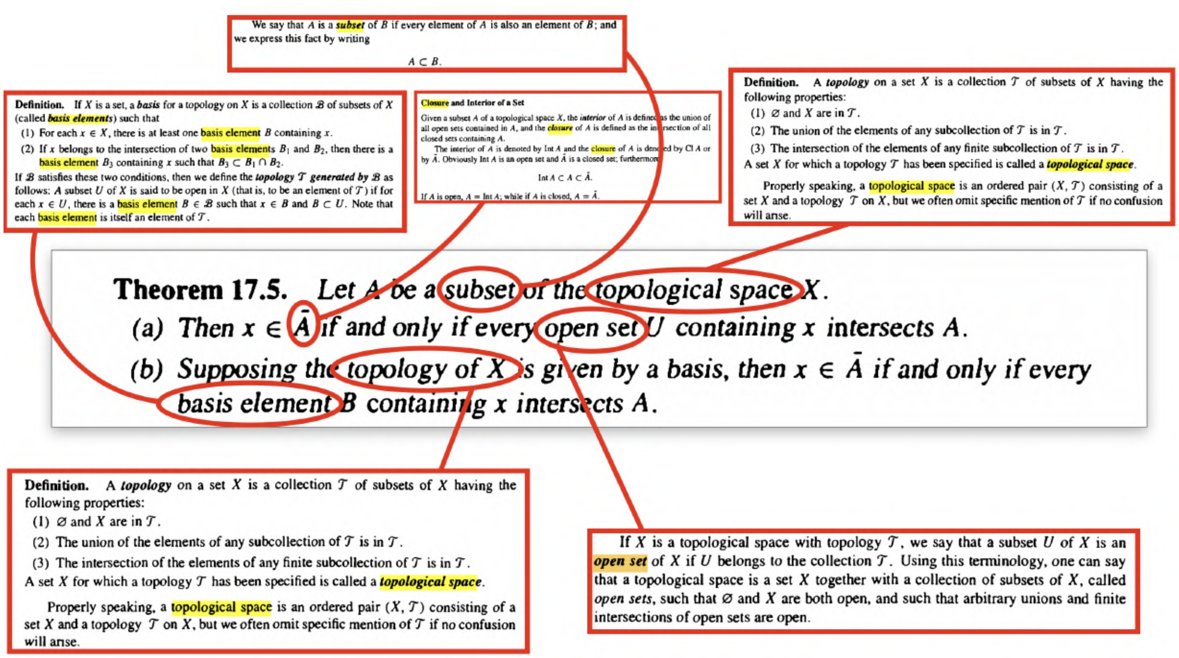
Exercises

1. Prove the following "laws of algebra" for $\mathbb{R}$, using only axioms (1)–(5):
(a) If $x + y = x$, then $y = 0$.
(b) $0 \cdot x = 0$. [*Hint:* Compute $(x + 0) \cdot x$.]
(c) $-0 = 0$.
(d) $-(-x) = x$.
(e) $x(-y) = -(xy) = (-x)y$.
(f) $(-1)x = -x$.
(g) $x(y - z) = xy - xz$.
(h) $-(x + y) = -x - y$; $-(x - y) = -x + y$.
(i) If $x \neq 0$ and $x \cdot y = x$, then $y = 1$.
(j) $x/x = 1$ if $x \neq 0$.
(k) $x/1 = x$.
(l) $x \neq 0$ and $y \neq 0 \Rightarrow xy \neq 0$.
(m) $(1/y)(1/z) = 1/(yz)$ if $y, z \neq 0$.
(n) $(x/y)(w/z) = (xw)/(yz)$ if $y, z \neq 0$.
(o) $(x/y) + (w/z) = (xz + wy)/(yz)$ if $y, z \neq 0$.
(p) $x \neq 0 \Rightarrow 1/x \neq 0$.
(q) $1/(w/z) = z/w$ if $w, z \neq 0$.
(r) $(x/y)/(w/z) = (xz)/(yw)$ if $y, w, z \neq 0$.
(s) $(ax)/y = a(x/y)$ if $y \neq 0$.
(t) $(-x)/y = x/(-y) = -(x/y)$ if $y \neq 0$.
2. Prove the following "laws of inequalities" for $\mathbb{R}$, using axioms (1)–(6) along with the results of Exercise 1:
(a) $x > y$ and $w > z \Rightarrow x + w > y + z$.
(b) $x > 0$ and $y > 0 \Rightarrow x + y > 0$ and $x \cdot y > 0$.
(c) $x > 0 \Leftrightarrow -x < 0$.
(d) $x > y \Leftrightarrow -x < -y$.
(e) $x > y$ and $z < 0 \Rightarrow xz < yz$.
(f) $x \neq 0 \Rightarrow x^2 > 0$, where $x^2 = x \cdot x$.
(g) $-1 < 0 < 1$
(h) $xy > 0 \Leftrightarrow x$ and y are both positive or both negative.
(i) $x > 0 \Rightarrow 1/x > 0$.
(j) $x > y > 0 \Rightarrow 1/x < 1/y$.
(k) $x < y \Rightarrow x < (x + y)/2 < y$.
3. (a) Show that if $\mathcal{A}$ is a collection of inductive sets, then the intersection of the elements of $\mathcal{A}$ is an inductive set.
(b) Prove the basic properties (1) and (2) of $\mathbb{Z}_+$.
4. (a) Prove by induction that given $n \in \mathbb{Z}_+$, every nonempty subset of $\{1, \ldots, n\}$ has a largest element.
(b) Explain why you cannot conclude from (a) that every nonempty subset of $\mathbb{Z}_+$ has a largest element.

Exercises

1. Give an example to show that if X is paracompact, it does not follow that for every open covering $\mathcal{A}$ of X, there is a locally finite *subcollection* of $\mathcal{A}$ that covers X.
2. (a) Show that the product of a paracompact space and a compact space is paracompact. [*Hint:* Use the tube lemma.]
(b) Conclude that S_Ω is not paracompact.
3. Is every locally compact Hausdorff space paracompact?
4. (a) Show that if X has the discrete topology, then X is paracompact.
(b) Show that if $f : X \to Y$ is continuous and X is paracompact, the subspace $f(X)$ of Y need not be paracompact.
5. Let X be paracompact. We proved a "shrinking lemma" for arbitrary indexed open coverings of X. Here is an "expansion lemma" for arbitrary locally finite indexed families in X.
Lemma. *Let $\{B_\alpha\}_{\alpha \in J}$ be a locally finite indexed family of subsets of the paracompact Hausdorff space X. Then there is a locally finite indexed family $\{U_\alpha\}_{\alpha \in J}$ of open sets in X such that $B_\alpha \subset U_\alpha$ for each α.*
6. (a) Let X be a regular space. If X is a countable union of compact subspaces of X, then X is paracompact.
(b) Show $\mathbb{R}^\infty$ is paracompact as a subspace of $\mathbb{R}^\omega$ in the box topology.
*7. Let X be a regular space.
(a) If X is a finite union of closed paracompact subspaces of X, then X is paracompact.
(b) If X is a countable union of closed paracompact subspaces of X whose interiors cover X, show X is paracompact.
8. Let $p : X \to Y$ be a perfect map. (See Exercise 7 of §31.)
(a) Show that if Y is paracompact, so is X. [*Hint:* If $\mathcal{A}$ is an open covering of X, find a locally finite open covering of Y by sets B such that $p^{-1}(B)$ can be covered by finitely many elements of $\mathcal{A}$; then intersect $p^{-1}(B)$ with these elements of $\mathcal{A}$.]
(b) Show that if X is a paracompact Hausdorff space, then so is Y. [*Hint:* If $\mathcal{B}$ is a locally finite closed covering of X, then $\{p(B) \mid B \in \mathcal{B}\}$ is a locally finite closed covering of Y.]

Of course the author defines all the terms here in previous pages, as part of that wall of definitions, but they do not build any intuition! There is no motivation for why they matter!



We say that A is a ***subset*** of B if every element of A is also an element of B; and we express this fact by writing
$$A \subset B.$$

Definition. If X is a set, a ***basis*** for a topology on X is a collection $\mathcal{B}$ of subsets of X (called ***basis elements***) such that
(1) For each $x \in X$, there is at least one basis element B containing x.
(2) If x belongs to the intersection of two basis elements B_1 and B_2, then there is a basis element B_3 containing x such that $B_3 \subset B_1 \cap B_2$.
If $\mathcal{B}$ satisfies these two conditions, then we define the ***topology $\mathcal{T}$ generated by $\mathcal{B}$*** as follows: A subset U of X is said to be open in X (that is, to be an element of $\mathcal{T}$) if for each $x \in U$, there is a basis element $B \in \mathcal{B}$ such that $x \in B$ and $B \subset U$. Note that each basis element is itself an element of $\mathcal{T}$.

Closure and Interior of a Set

Given a subset A of a topological space X, the ***interior*** of A is defined as the union of all open sets contained in A, and the ***closure*** of A is defined as the intersection of all closed sets containing A.

The interior of A is denoted by Int A and the closure of A is denoted by Cl A or by $\bar{A}$. Obviously Int A is an open set and $\bar{A}$ is a closed set; furthermore
$$\text{Int } A \subset A \subset \bar{A}.$$
If A is open, $A = \text{Int } A$; while if A is closed, $A = \bar{A}$.

Definition. A ***topology*** on a set X is a collection $\mathcal{T}$ of subsets of X having the following properties:
(1) $\varnothing$ and X are in $\mathcal{T}$.
(2) The union of the elements of any subcollection of $\mathcal{T}$ is in $\mathcal{T}$.
(3) The intersection of the elements of any finite subcollection of $\mathcal{T}$ is in $\mathcal{T}$.
A set X for which a topology $\mathcal{T}$ has been specified is called a ***topological space***.

Properly speaking, a topological space is an ordered pair $(X, \mathcal{T})$ consisting of a set X and a topology $\mathcal{T}$ on X, but we often omit specific mention of $\mathcal{T}$ if no confusion will arise.

Theorem 17.5. *Let A be a subset of the topological space X.*
(a) Then $x \in \bar{A}$ if and only if every open set U containing x intersects A.
(b) Supposing the topology of X is given by a basis, then $x \in \bar{A}$ if and only if every basis element B containing x intersects A.

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If X is a topological space with topology $\mathcal{T}$, we say that a subset U of X is an ***open set*** of X if U belongs to the collection $\mathcal{T}$. Using this terminology, one can say that a topological space is a set X together with a collection of subsets of X, called *open sets*, such that $\varnothing$ and X are both open, and such that arbitrary unions and finite intersections of open sets are open.

You would've definitely taken a very, very long time to properly digest all the theorems and would have probably gone on without completely understanding everything.

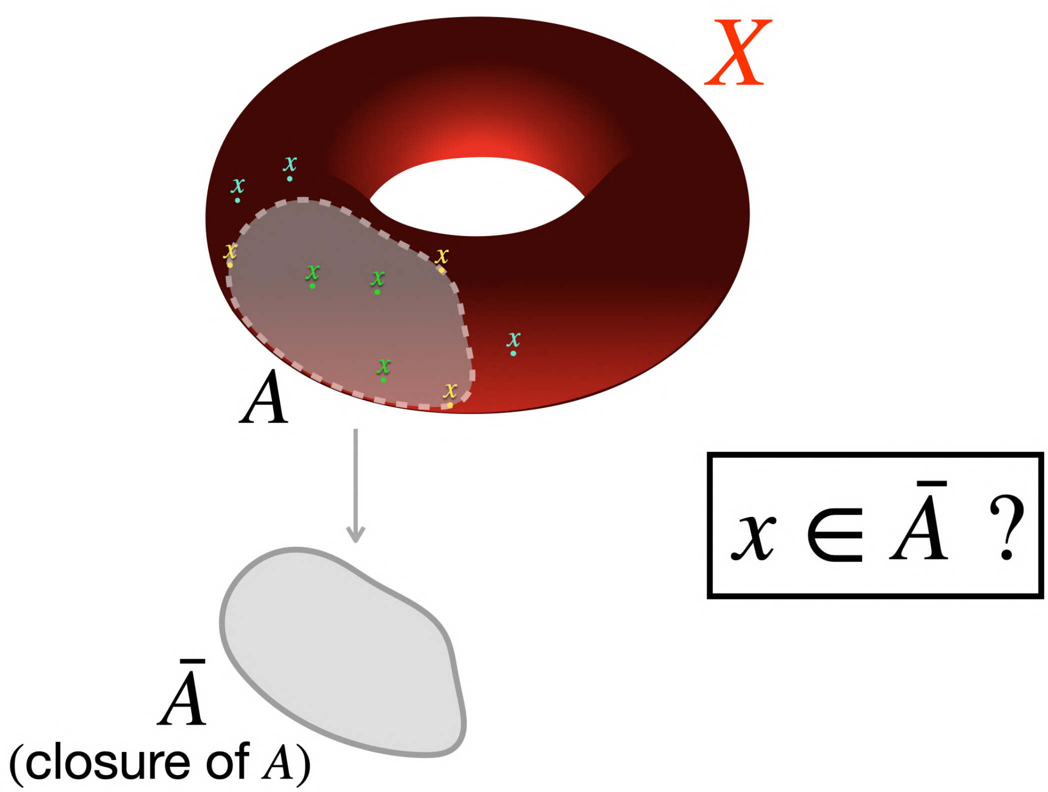
I also want to say that this is not a personal criticism to the author. Not at all! From a rigorous point of view, the book is amazing! As far as rigor is concerned, it is very beginner-friendly. It does define most things, and it's very complete. But learning math is *more than rigor*, it requires building *intuition first*, showing many *concrete examples*, and only then introducing these concepts in a formal way. So all I am saying is that, I think we can do a better job than that. We can publish better books than we currently do.

Now, let's study this theorem, but the **right way** this time:

The Right Way

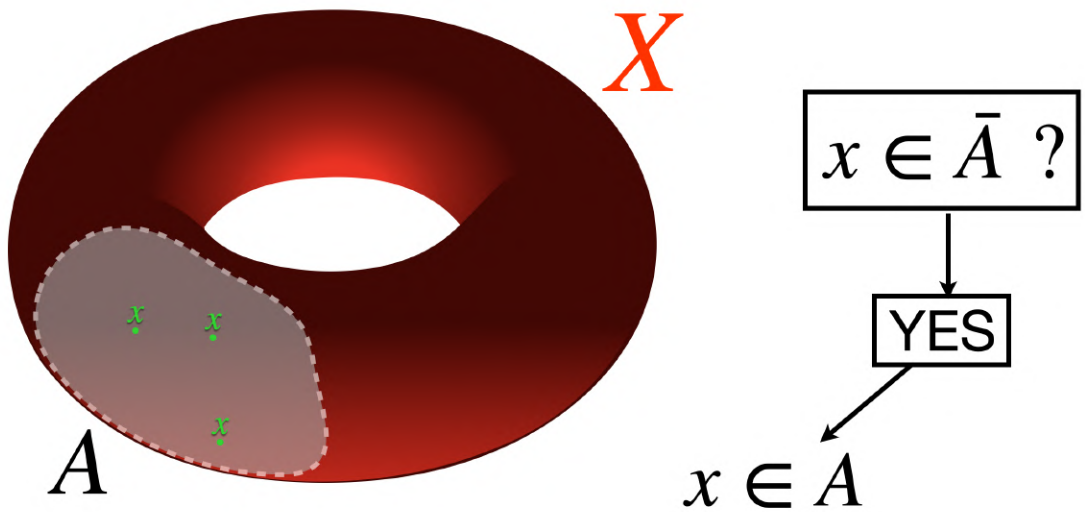
what is its intent?

Before even introducing the theorem, we need to answer the following questions: *"What is its intent?! What is the motivation for it? What are we trying to accomplish?"* Our goal is to understand when a point x is in the *closure* of a set A in a topological space X.



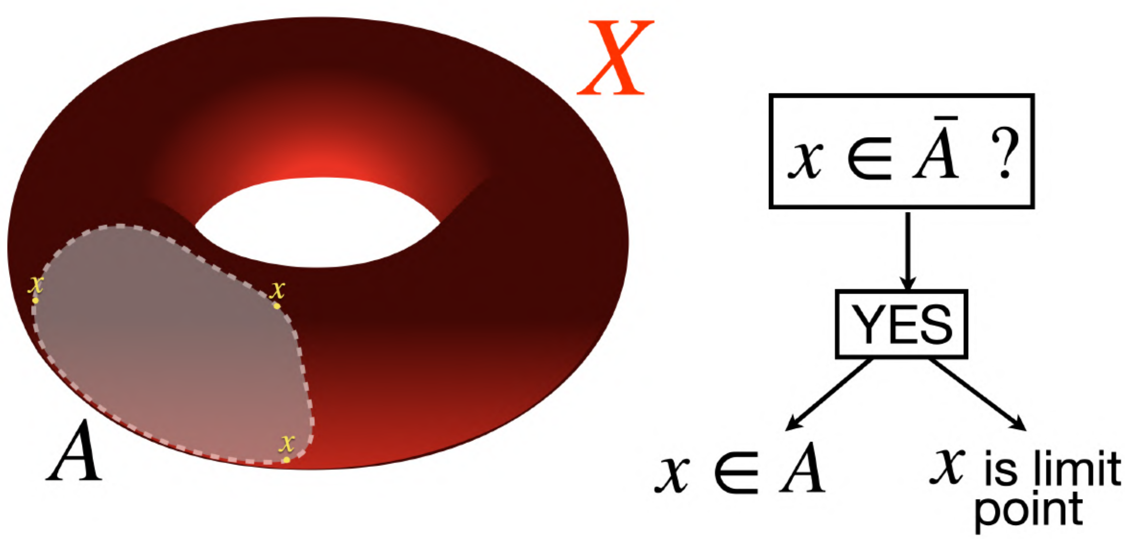
The closure of A (i.e. $\bar{A}$) includes all the points that are either:

$\rightarrow$ in A $\quad (x \in A)$



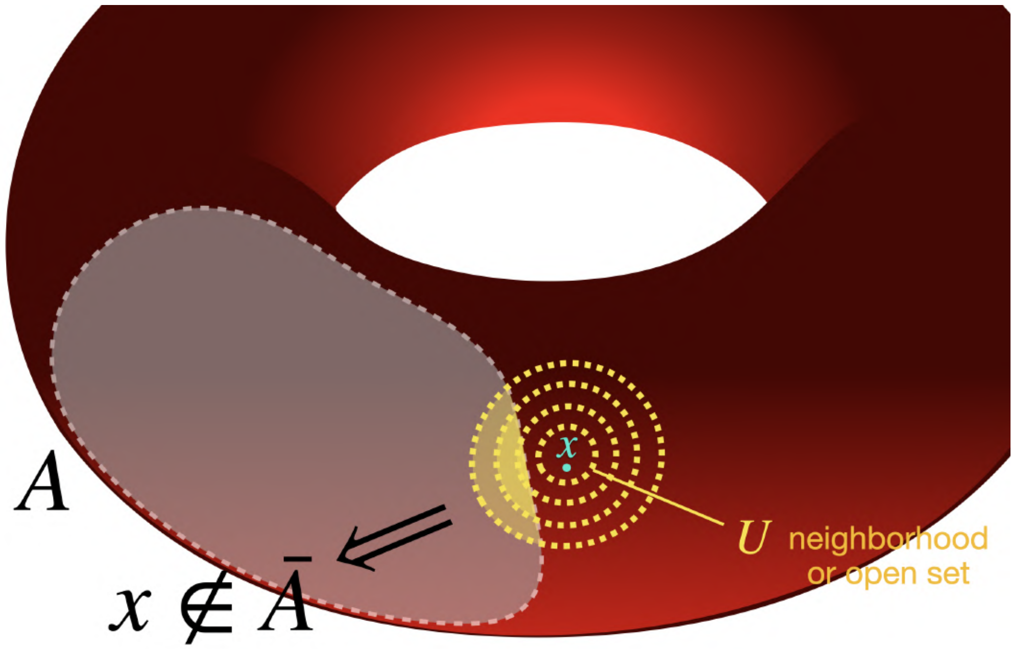
, or

$\rightarrow$ are *limit points* of A (meaning that you can't get arbitrarily close to it without bumping into A).

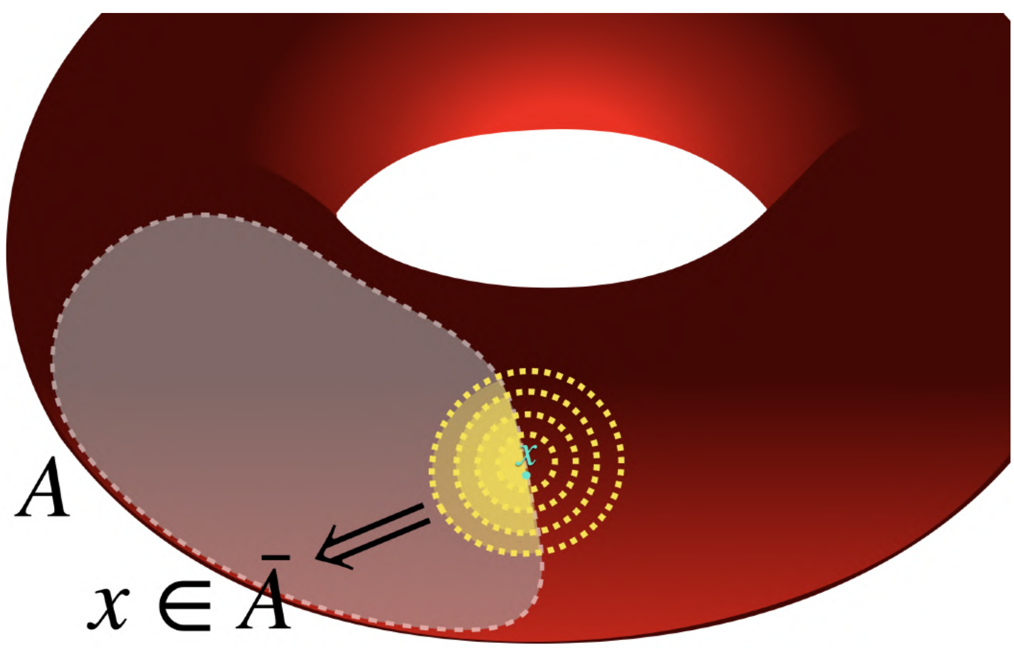


So, intuitively, what the first part of the theorem says is that:

If you can create an open neighborhood around a point x that avoids intersecting A, then x is NOT close enough to A, and therefore $x \notin \bar{A}$.



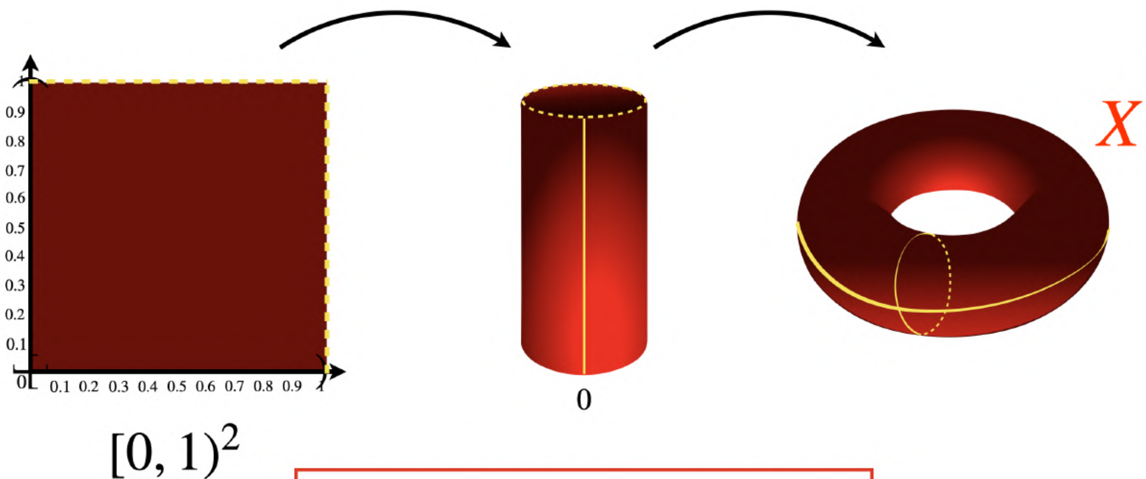
On the other hand, if every open set (or neighborhood U) around a point x (no matter how small you make it) still intersects A somewhere, then x is close enough to A and we say that $x \in \bar{A}$.



Let's see a concrete example:

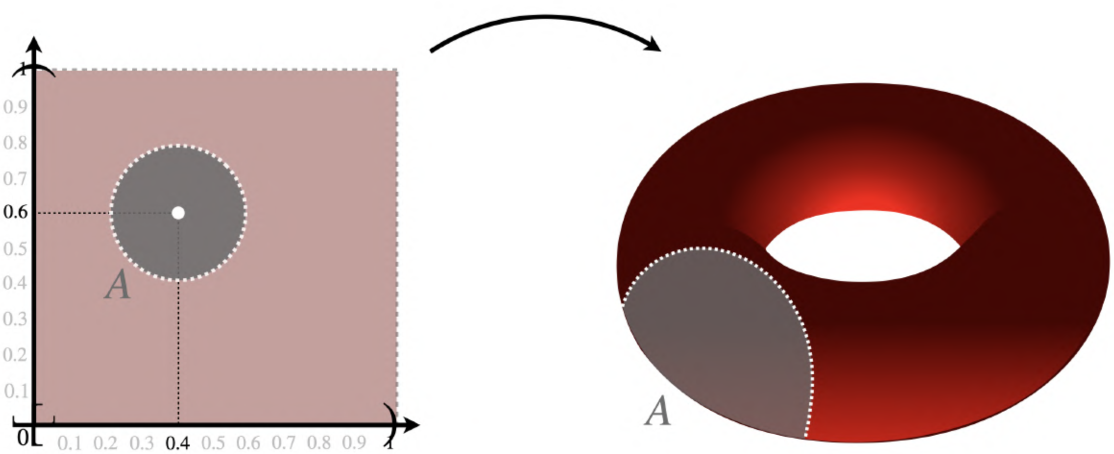
Concrete Example

The topological space X will be the 2-torus $X = \mathbb{T}^2$, which can be described as the unit square $[0\,,\,1)^2$, where opposite edges are identified.



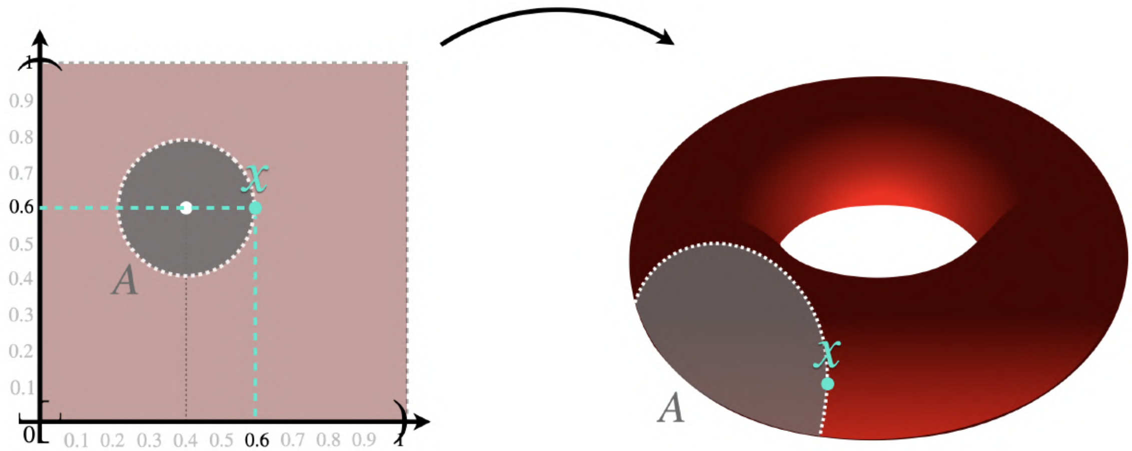
$$X = \mathbb{T}^2 = \mathbb{R}^2 \mod \mathbb{Z}^2$$

Let A be an open disk of radius 0.2 centered at $(0.4, 0.6)$.



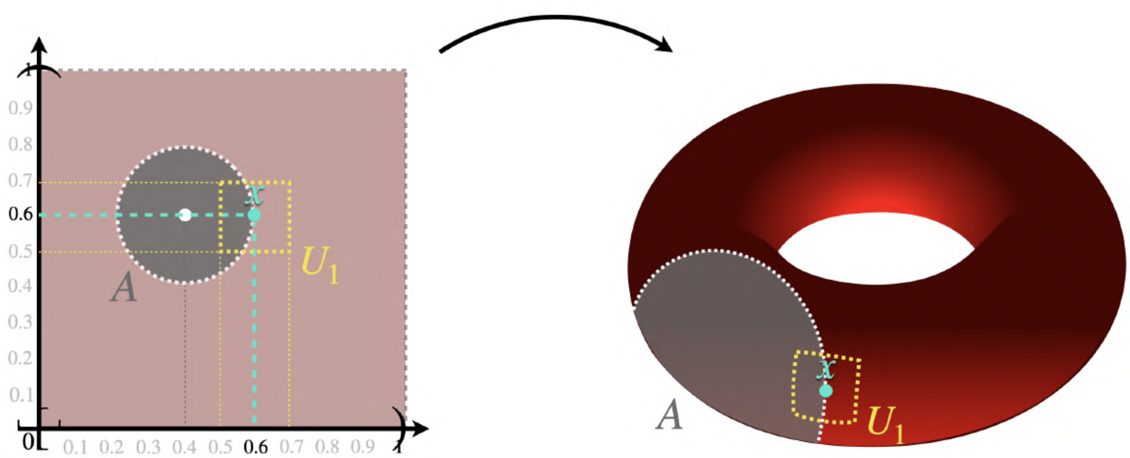
$$A = \left\{ (x, y) \in \mathbb{T}^2 \,\middle|\, \sqrt{(x-0.4)^2 + (y-0.6)^2} < 0.2 \right\}$$

Now, we choose a point $x = (0.6, 0.6)$, and we check whether it's part of the closure of A, or not.



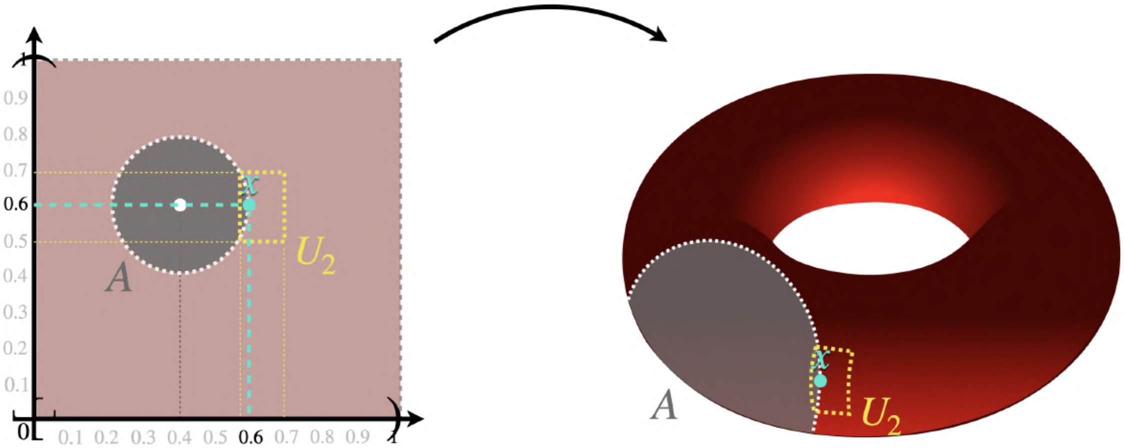
$$\Longrightarrow x \in \bar{A}?$$

Let's pick a neighborhood U_1 around x, defined as a square $(0.5, 0.7) \times (0.5, 0.7)$ of side length 0.2. As you can see $U_1 \cap A \neq \emptyset$.



$$U_1 \cap A \neq \emptyset$$

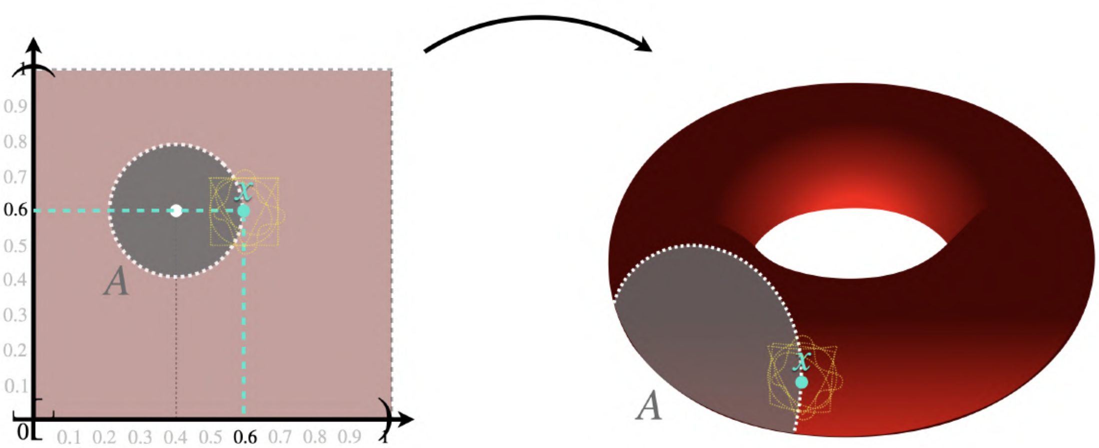
Now, pick a smaller neighborhood U_2 around x, defined as a rectangle $(0.59, 0.7) \times (0.5, 0.7)$. We still have that $U_2 \cap A \neq \emptyset$.



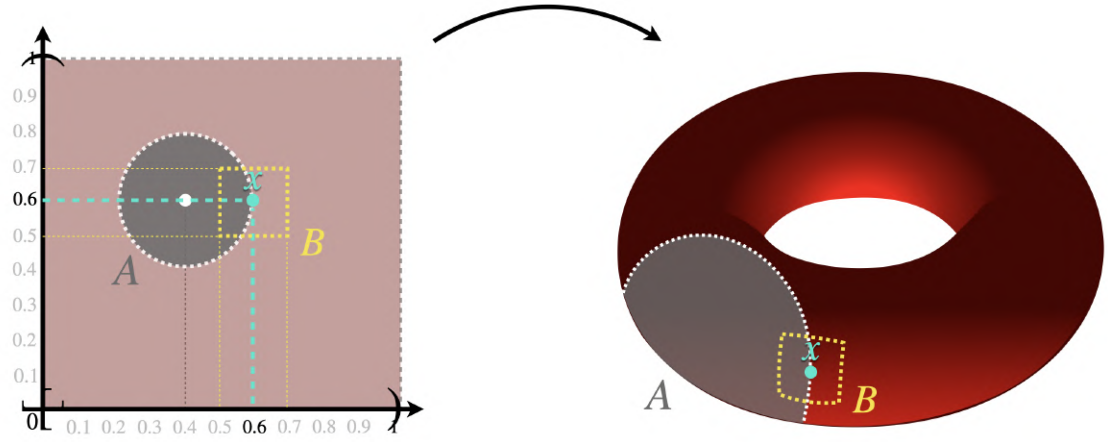
$$U_2 \cap A \neq \varnothing$$

Of course, it's easy to convince ourselves that no matter how small the neighborhood U is, we always get that $U \cap A \neq \varnothing$. However, technically, if we want to use the result of point **(a)** of this theorem, we need to show that for every neighborhood U around x, and not only rectangular ones, but all possible shapes of open sets, which is kind of impractical...

(a) *Then* $x \in \bar{A}$ *if and only if every open set* U *containing* x *intersects* A.

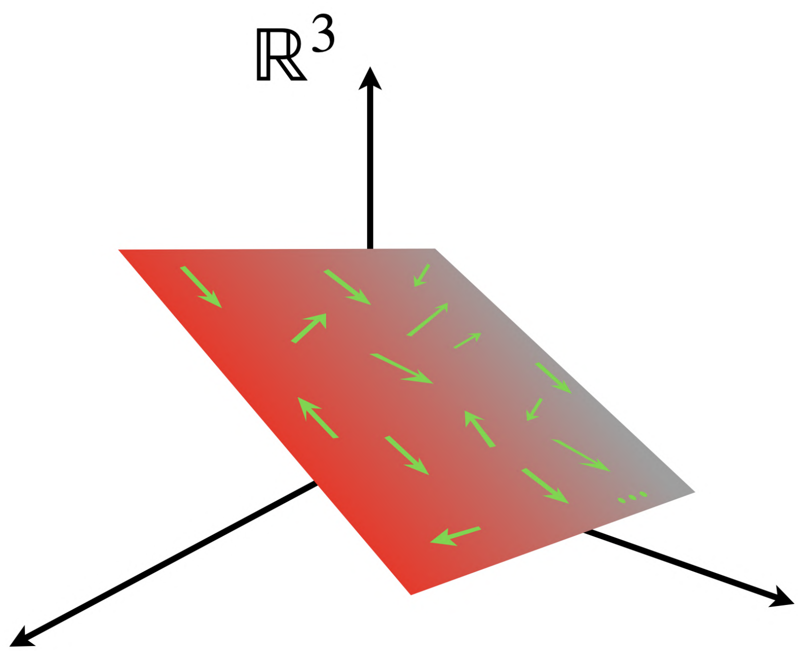


And that's why we have point **(b)** of the theorem.

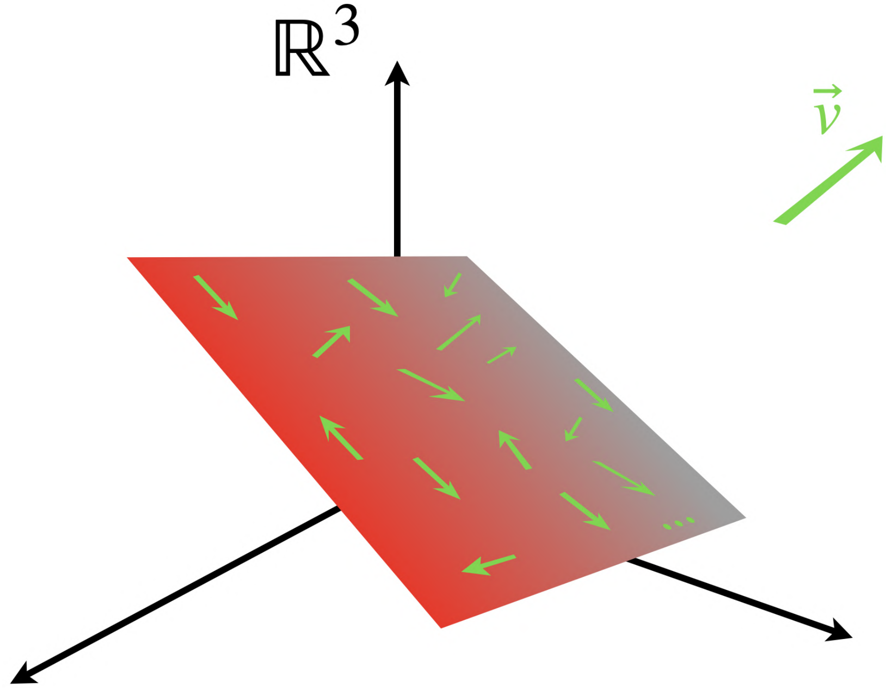


(b) Supposing the topology of X is given by a basis, then $x \in \bar{A}$ if and only if every basis element B containing x intersects A.

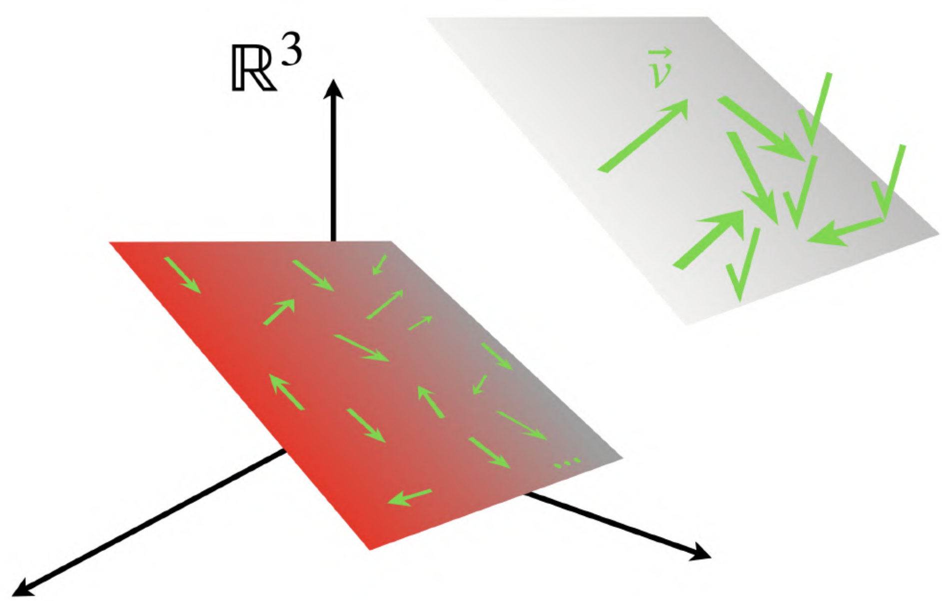
First, we must define what a basis is for a topology. Let's make an analogy from linear algebra:



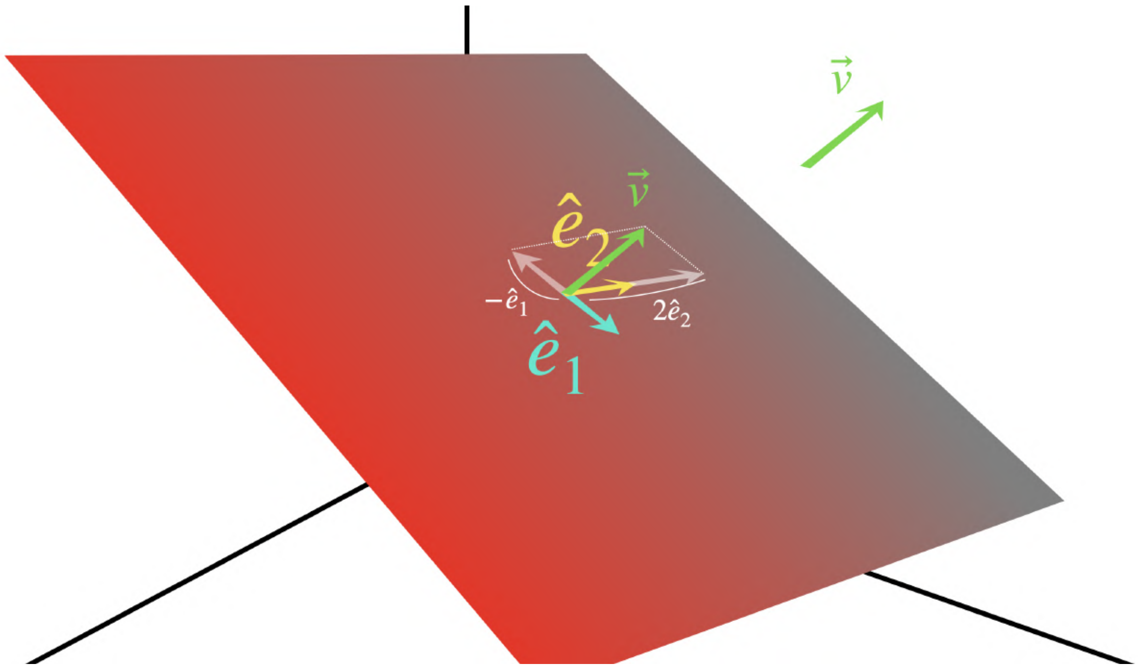
Imagine a $2D$ plane inside a $3D$ space, like the xy-plane in $\mathbb{R}^3$. This plane contains infinitely many vectors.



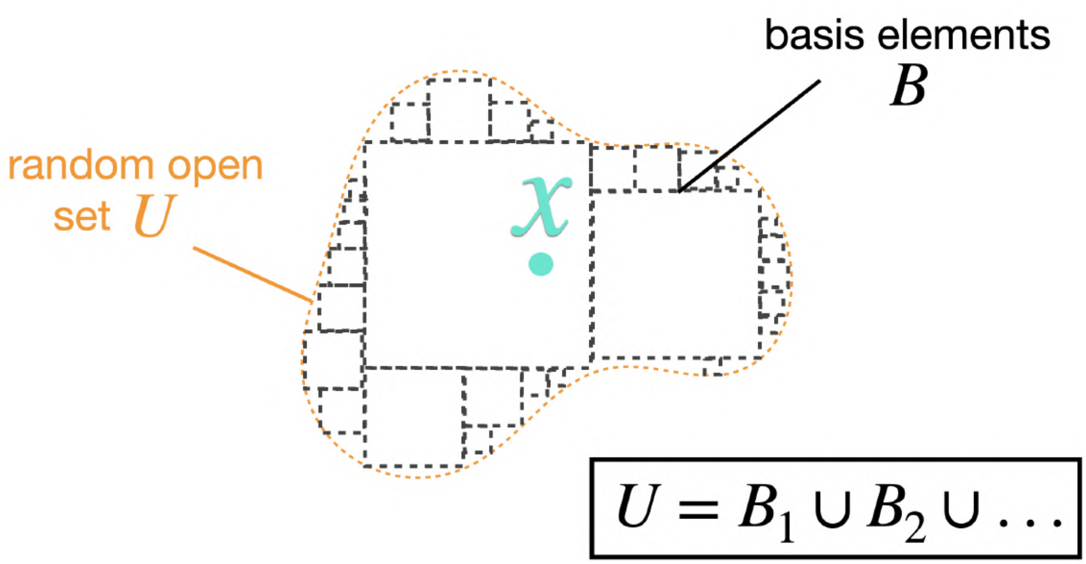
But if you want to check whether some vector $\vec{v} \in \mathbb{R}^3$ is parallel to the plane, do you need to test it against every vector in the plane?



No! You only need to check whether $\vec{v}$ lies in the span of just two basis elements (i.e. basis vectors), say $\hat{e}_1$ and $\hat{e}_2$. If it's a linear combination of those two, then it lies in the plane.

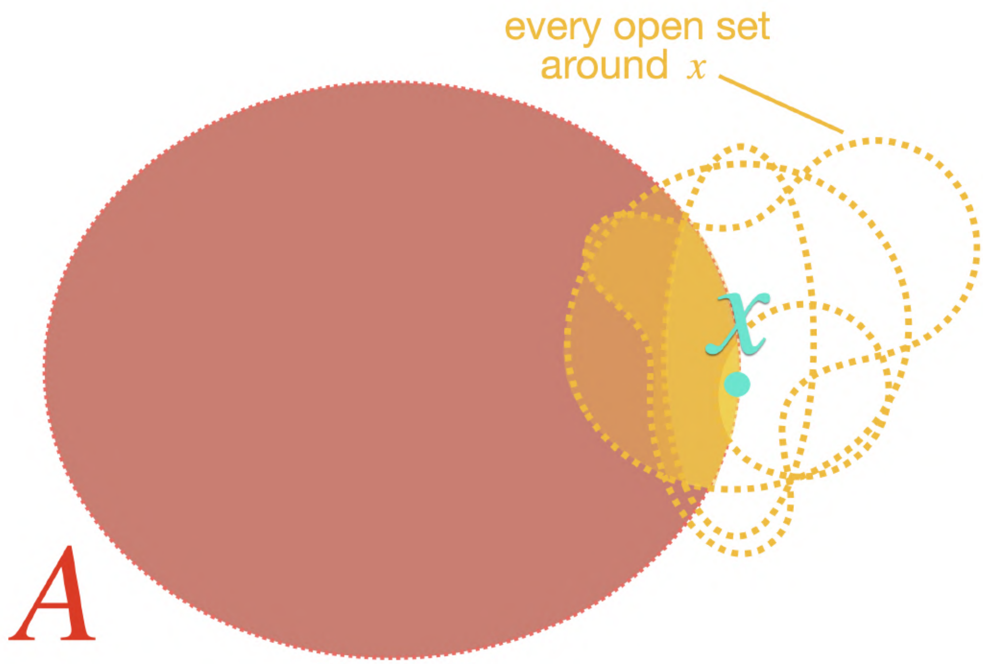


In an analogous way, in a topological space, there are infinitely many open sets, but some properties can be checked just against basis elements for a topology, and they will automatically be true for every open set in the space.



A *basis* for a topology is a collection of "basis open sets" that we use to build all other open sets via unions.

The essence of point **(b)** in the theorem is that *"if every basis element around a point intersects A, then every open set around that point (which is built from those basis elements) will also intersect A."*



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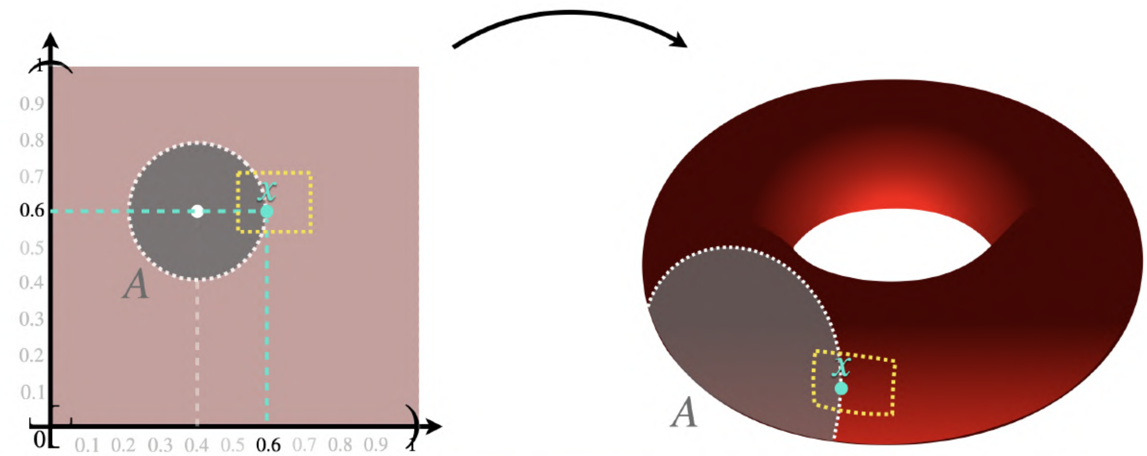
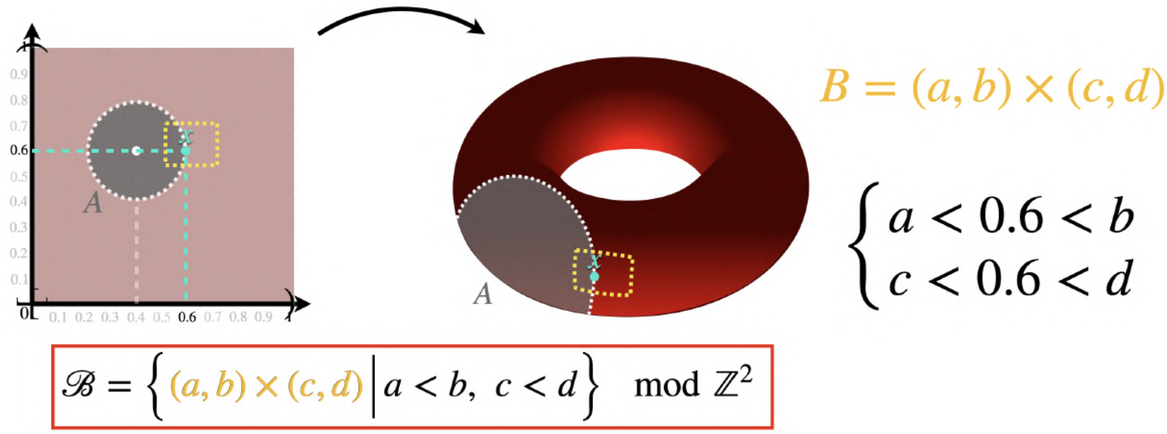
Going back to our example, we can use the standard basis (the "fancy B" below) for $\mathbb{T}^2$:

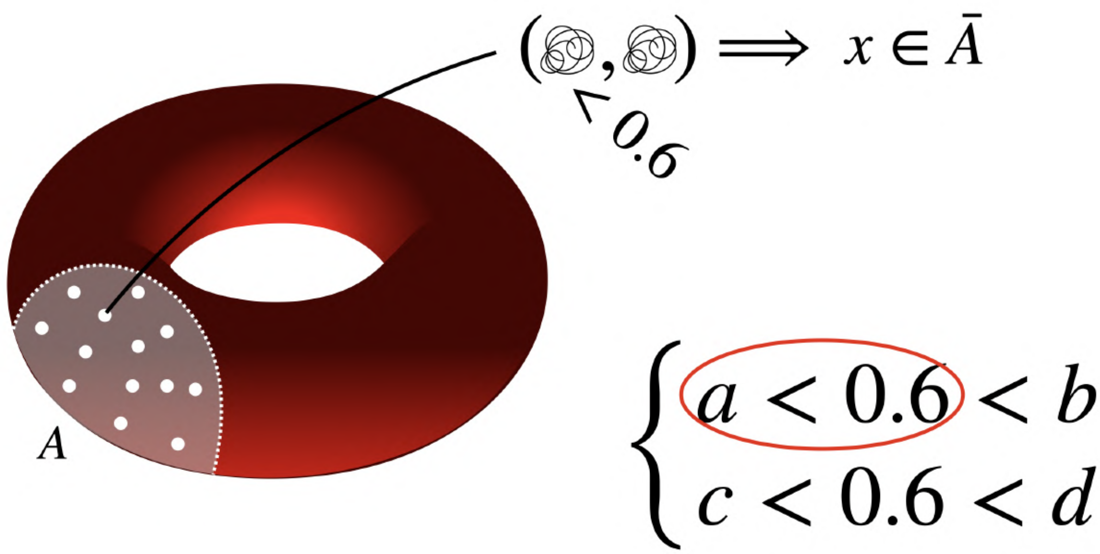


$$\mathscr{B} = \left\{ (a,b) \times (c,d) \,\middle|\, a < b,\ c < d \right\} \mod \mathbb{Z}^2$$

These are open rectangles as you can see. Take $x = (0.6, 0.6)$, just as before. In order for a basis element $B = (a,b) \times (c,d)$ to contain x, it must satisfy these 4 inequalities:



In particular, any such rectangle must have $a < 0.6$. But all points in the set A have first coordinates < 0.6 as well. So they always overlap.



Try to reread the rigorous theorem and its proof now. I am sure that you will find it to be waaaay easier than before.

Theorem 17.5. *Let A be a subset of the topological space X.*

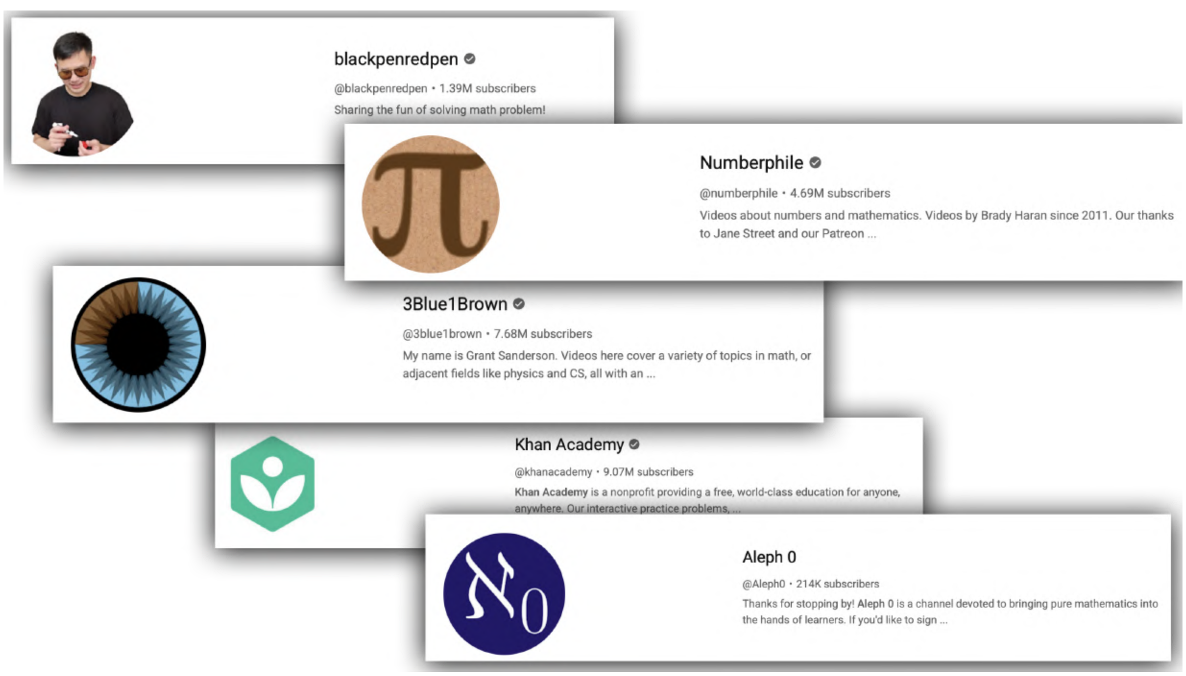
(a) *Then $x \in \bar{A}$ if and only if every open set U containing x intersects A.*

(b) *Supposing the topology of X is given by a basis, then $x \in \bar{A}$ if and only if every basis element B containing x intersects A.*

The Method

I mean, I am sure that the author himself could've done a better job than me at explaining the theorem in an intuitive way. The reason why he, and most math authors, don't do that, is a mystery to me...

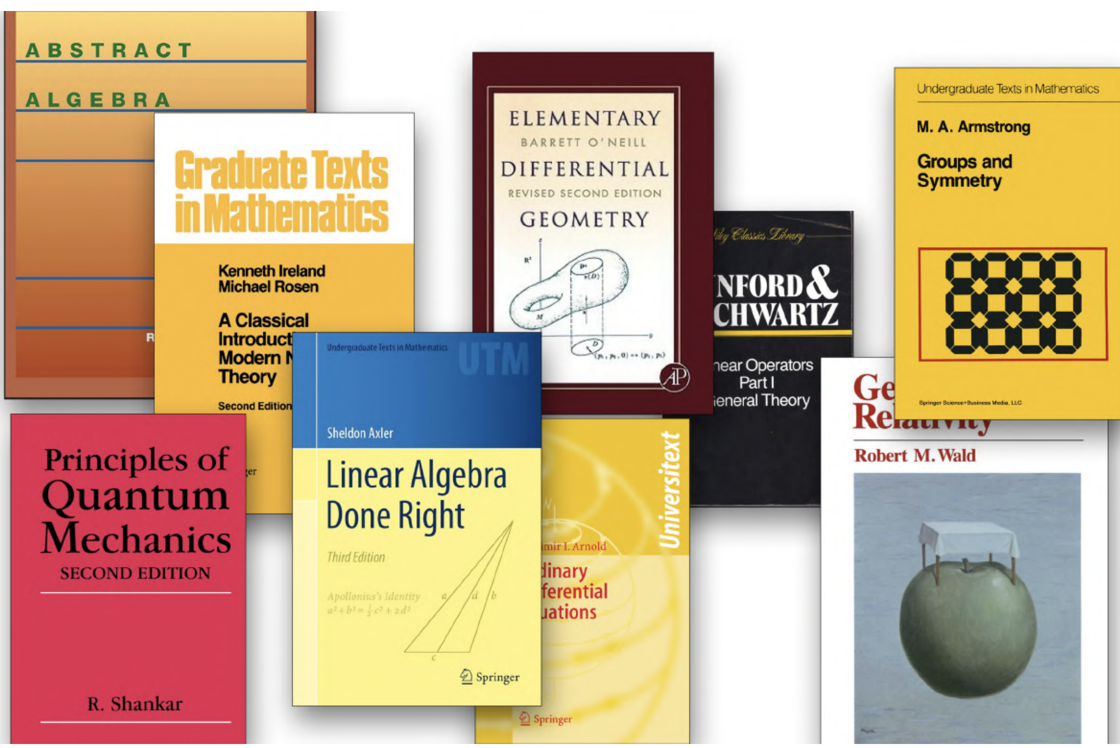
Look, I am not trying to flex here, I am just trying to say that there is a lot of content on the internet (including other YouTube channels) that do a much better job at explaining math than 99% of advanced math books. This is just a fact.



The proper way of learning not only point-set topology, but any math is: **1. Intuition**, **2. Concrete examples**, **3. Rigor**, **4. Practice** with exercises (which this particular book thankfully provides, but of course with no solutions… so, you can't really know if you solved them correctly… Classic.)

1. intuition
2. concrete examples
3. rigor
4. practice (with exercises)

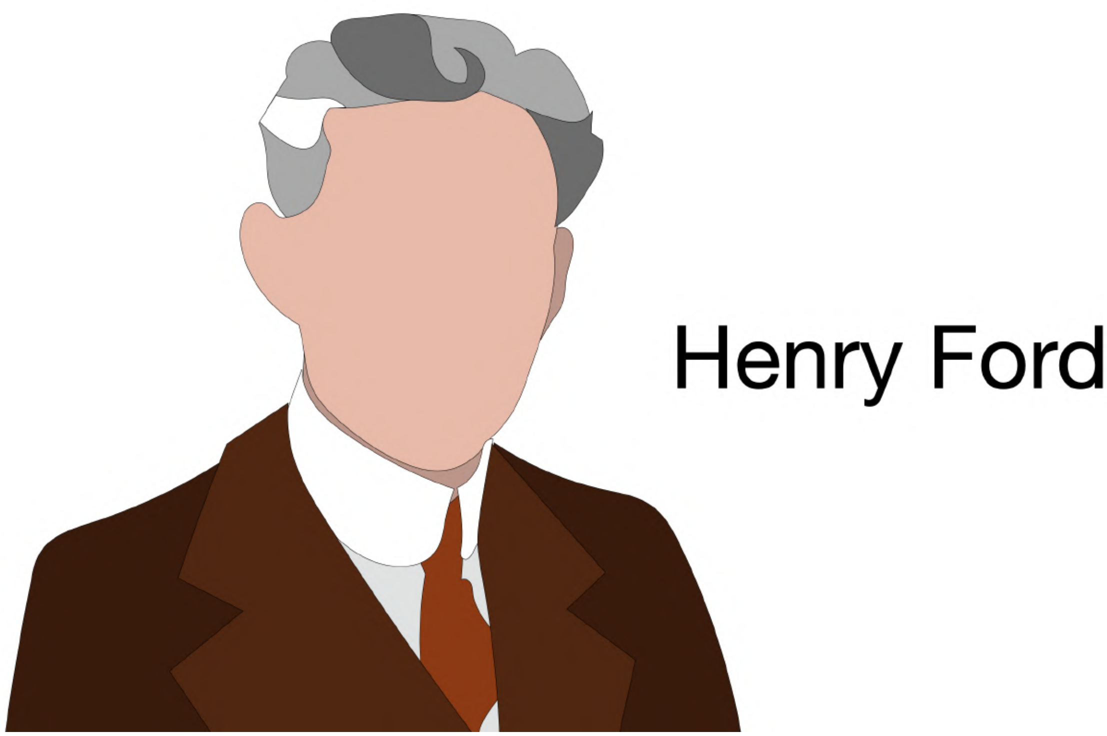
Now, imagine learning all the theorems, proofs and results in your favorite books the same way we studied this theorem here today! Wouldn't it make your life much easier?!



I am not saying that you cannot learn from these books, of course you're going to learn from these books. In fact, it would not make sense if you couldn't, since all professors and researchers nowadays learned from these books. I am just saying that they are not **optimal**, from the pedagogical point of view. Spitting out definitions, and theorems, without building intuition first, is a lazy way of exposing any subject. Mathematicians could do better, and dedicate more time to their explanation, rather than putting all of the work on the reader. It's already hard to try and learn new things in mathematics. It is much harder, when you have to decipher a wall of definitions, and figure out why we are even doing this??

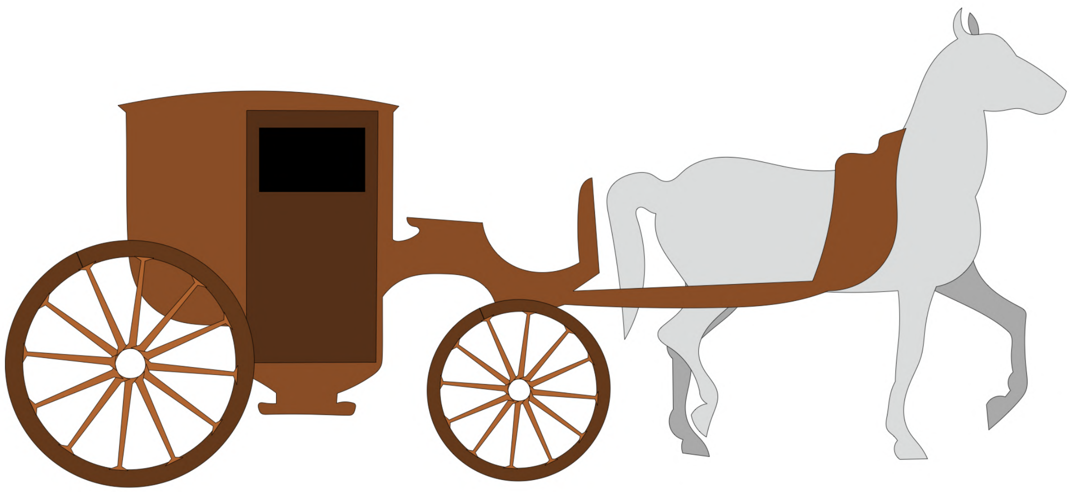
Conclusion

You know, *Henry Ford* didn't invent the car, but he was the one who made it affordable, and accessible to the masses.



There's a well-known quote, which is often repeated in tech and innovation circles, and it's usually misattributed to him:

“If I had asked people what they wanted, they would have said faster horses.”



Whether he said it or not isn’t really the point. What matters is the idea behind it. People tend to think in terms of what they already know, not in terms of what’s possible. Nobody was imagining a world without horses. They just wanted to improve the things they already had. Because that’s what they were used to. People rarely ask for change, even when the alternative is clearly better. And honestly, higher math education has been stuck in the same place for decades. This rigid format (definition, theorem, proof, repeat), without building any intuition first, has become so normalized that most people can’t even imagine another way to learn math. It’s robotic. But it doesn’t have to be this way.

The point that I am trying to get across is that in general, we can write better math books. We can design learning experiences that are deeper, faster, and way more human, especially now, with all the tools we have: AI, YouTube, online forums, visualization software, interactive platforms. We don’t need to keep dragging people through decade-old formats just because that’s how we always did it.

Change doesn't happen by accident. It requires conscious effort, and that's exactly what we want to do with our YouTube channel over many years to come.

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