Notice that the fraction $\frac{i}{\sqrt{i}}$ can be rationalized by multiplying the numerator and the denominator by $\sqrt{i}$. Now we can simply cancel out these 2 imaginary units on the top and on the bottom and this gives us $\sqrt{i}$. So we found our first equal: $\frac{i}{\sqrt{i}} = \sqrt{i}$.

Notice also that

$$\frac{1}{\sqrt{i}} = \frac{\sqrt{1}}{\sqrt{i}} = \sqrt{\frac{1}{i}} = \sqrt{\frac{1}{\sqrt{-1}}} = \sqrt{\frac{\sqrt{1}}{\sqrt{-1}}} = \sqrt{\sqrt{\frac{1}{-1}}} = \sqrt{\sqrt{-1}} = \sqrt{i} .$$

Thus, we found our second equation. It turns out that these 2 equations are the same. So, we can say that $\frac{i}{\sqrt{i}} = \frac{1}{\sqrt{i}}$. But wait a second, this means that if we cancel out these 2 terms in the denominator we get $i = 1$.

Thanks…

No! This is wrong! But why is it wrong?!?!?! What was our mistake here?! Well, let's replay the scene:

$$\frac{1}{\sqrt{i}} = \frac{\sqrt{1}}{\sqrt{i}} = \sqrt{\frac{1}{i}} = \sqrt{\frac{1}{\sqrt{-1}}} = \sqrt{\frac{\sqrt{1}}{\sqrt{-1}}} \neq \sqrt{\sqrt{\frac{1}{-1}}} = \sqrt{\sqrt{-1}} = \sqrt{i}$$

This is not true! So, actually our second equation is not true. But why? Well, the answer is that the property $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ is valid **if and only if** $a \geq 0$ and $b > 0$. So the problem is that we cannot apply this property for $\sqrt{-1}$. And that's where the contradiction appears.